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MATHS
PRIMARY LEVEL
UNISWA
SWAZILAND
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Grades 7-9 Priority Topics

- Place Value
- Addition and Subtraction with regrouping
- Fractions, decimals and percents
- Converting between SI units
- Financial Maths

Place Value, Common Fractions, and Decimal Fractions

1. We use a decimal place value system where whole numbers are written so that the position of digits in numbers determines what they represent. Digits in whole numbers can be ones, tens, hundreds, thousands, ten thousands, and so forth. For example, the digit 5 in the number 1 354 is said to be in the hundreds place. Altogether, we have "1 thousand, 3 hundreds, 5 tens, and 4 ones."
   (a) In the number 214 567, what digit is in the ten thousands place?
   (b) In the number 214 567, the digit 6 is said to be in which place?
2. Identify the number in the the thousands place in each of the following:
   (a) 345 234
   (b) 1 233 235
   (c) 56 679
3. If we try to *round* a number to the nearest of a given place value, we must look at the number in the place to the right of the place value. Explain why we would say that $11, 12, 13$, and 14 would round *down* to 10, but $15, 16, 17, 18, 19$ round *up* to 20.
4. (Continuation) Round the following to the nearest ten:
   (a) 34
   (b) 66
   (c) 87
5. Round the following to the nearest thousand:
   (a) 345 234
   (b) 1 233 235
   (c) 56 679
6. Round the following to the nearest ten thousand:

   (a) 345 234
   (b) 1 233 235
   (c) 56 679

7. A *decimal fraction* is a fraction whose denominator is a number such as 10, 100, 1000, etc. (in other words, a power of ten). We can express the common fraction $\frac{1}{10}$ as a decimal fraction as 0.1. The digit after the decimal mark is said to be in the *tenths place*. We would express $\frac{2}{10}$ as 0.2. How would you write 0.9 as a common fraction?

8. A pizza is cut into 10 slices and 3/5 of it is left.
   (a) How many slices are left?
   (b) Express the fractional part that is left over as a fraction with a denominator of 10.

9. (Continuation) How many tenths are in your answer to the previous problem? How would you express your answer to the previous problem as a decimal fraction?

10. Can you convert $\frac{3}{4}$ into a common fraction with an integral numerator and a denominator of 10? How about into a common fraction with an integral numerator and a denominator of 100?

11. (Continuation) Explain why the previous answer could be written as $\frac{7}{10} + \frac{5}{100}$. When a number is expressed as a decimal fraction, the digit that is to the left of the tenths place is said to be in the *hundredths place*. How do you think you could express $\frac{3}{4}$ in decimal form? What digit is in the tenths place? What digit is in the hundredths place?

12. Express $\dfrac{24}{50}$ as decimal fraction.

13. Express the following common fractions as decimal fractions:
   (a) $\dfrac{3}{25}$
   (b) $\dfrac{2}{8}$
   (c) $\dfrac{5}{2}$

14. Farah and 6 of her friends were playing one day when they discovered an envelope with nine $1000 bills, eight $100 bills, two $10 bills and eight $1 bills.
   (a) How much total money did they find?
   (b) What do you think would be a fair way to split the money between them? How much would each person get?

15. (Continuation) Use *long division* to divide 9828 by 7. How does this relate to the previous problem?

16. Use long division to divide 727096 by 8.
17. Use long division to divide 727096 by 53.
18. A mother has E250.00 to spend on her 8 children.
    (a) Split E240.00 between the 8 children. How much will each child get? How many emalangeni will she have left over?
    (b) Split the remaining emalangeni between the children with remainder. How many will each child get? How many will be left over
    (c) Change the remainder emalangeni for 10-cent coins (tenths of a lilangeni). How many 10-cent coins will she have?
    (d) Split the 10-cent coins between the 8 children with remainder. How many would each child get? How many 10-cent coins will there be left over?
    (e) Change those 10-cent coins (tenths of a lilangeni) into 1-cent coins (hundredths of a lilangeni). How many 1-cent coins are there now? Can those be divided evenly between the 8 children?
    (f) In total, how much can the mother spend on each child?
19. (Continuation) Represent $\frac{250}{8}$ as a decimal fraction.
20. Explain how you can represent a common fraction into a decimal fraction. Demonstrate your procedure by representing $\frac{24}{50}$ as a decimal fraction.
21. Whole number portions of decimal numbers are represented by the digits to the left of the decimal mark. As we move further left, every number place gets 10 times bigger. As we move further right, every number place gets 10 times smaller (one tenth as large). The word "decimal" means "based on ten."
    (a) We know that the position to the right of the decimal mark is the tenths place and the one to the right of that is the hundredths. What is the name of the position to the right of the hundredths place? What about to the right of that?
    (b) What digit is in the millionths place in $12\,406\,927,357\,235\,138$?
    (c) Express $0.357\,235\,138$ as a common fraction.
22. Represent the following common fractions as decimal fractions:
    (a) $\frac{501}{6}$ (b) $\frac{13}{16}$ (c) $\frac{5}{8}$ (d)$1\frac{4}{5}$
23. (Continuation) What is the value in the hundredths place in each of the above decimal representations? Approximate each to the nearest tenth.
24. Represent the following decimal fractions as common fractions in lowest terms:
    (a) 0.52
    (b) 3.125
    (c) 435.23
25. Represent $124\,863.235$ as a common fraction.

Challenge Problems

26. Carton A of orange juice contains 24 ounces of orange concentrate and 40 ounces of water. Carton B contains 15 ounces of orange concentrate and 30 ounces of water.
    (a) What fraction of the juice in Carton A is orange concentrate?
    (b) What fraction of the juice in Carton B is orange concentrate?
    (c) Which carton has a stronger orange flavor?
27. Before you are able to take a bite of your new chocolate bar, a friend comes along and takes 1/4 of the bar. Then another friend comes along and you give this person 1/3 of what you have left. Make a diagram that shows the part of the bar left for you to eat.
28. (Continuation) Later you have another chocolate bar. This time, after you give away 1/3 of the bar, a friend breaks off 3/4 of the remaining piece. What part of the original chocolate bar do you have left? Answer this question by drawing a diagram.

Percents

1. Write the following fractions as equivalent fractions with a denominator of 100. How many hundredths do you have in each case?
   (a) $\frac{6}{50}$ (b) $\frac{3}{4}$ (c) $\frac{17}{25}$ (d) $\frac{1}{2}$
2. Write the following fractions as equivalent (but improp. rer) fractions with a denominator of 100. How many hundredths do you have in each case?
   (a) $\frac{11}{10}$
   (b) $\frac{7}{5}$
3. (Continuation) A *percent* is a fraction is a denominator of 100. Since $\frac{19}{20}$ can be written as $\frac{95}{100}$, or 95 hundredths, we say that "$\frac{19}{20}$ is 95%" or "19 is 95% of 20." Express the following fractions as percents.
   (a) $\frac{6}{50}$ (b) $\frac{3}{4}$ (c) $\frac{17}{25}$ (d) $\frac{1}{2}$ (e) $\frac{11}{10}$ (f) $\frac{7}{5}$
4. (Continuation) Answer the following:
   (a) 6 is what percent of 50?
   (b) 3 is what percent of 4?
   (c) 17 is what percent of 25?
   (d) 1 is what percent of 2?
   (e) 11 is what percent of 10?
   (f) 7 is what percent of 5?

5. Express the following decimals as fractions with a denominator of 100:
   (a) 0.13
   (b) 0.04
   (c) 0.98
   (d) 1.35
   (e) 2.03
6. (Continuation) Express the above decimals above as percents.
7. Before you are able to take a bite of your new chocolate bar, a friend comes along and takes 1/4 of the bar. Then another friend comes along and you give this person 1/3 of what you have left. Make a diagram that shows the part of the bar left for you to eat.
8. (Continuation) What percentage of the original chocolate bar did the first friend take? What percentage of the original chocolate bar did the second person take?
9. A football team has started its season badly, winning 2 games, losing 8, and tying none. The team will play a total of 25 games this season.
   (a) What percentage of the ten games played so far have been wins?
   (b) Starting with its current record of 2 win and 8 losses, what will the cumulative winning percentage be if the team wins the next 10 games in a row?
   (c) Starting with its current record of 2 win and 8 losses, how many games in a row must the team win in order for its cumulative winning percentage to reach at least 80%?
   (d) Suppose that the team wins nine of its remaining 15 games. What is its final winning percentage?
   (e) Is it possible for the team to have a final winning percentage of 95%? Explain.
10. The prices at a store rose by 20%. By how much did the price of a E100 shirt rise? What is the new price of the shirt?
11. (Continuation) What is 20% of 100? What about 200?
12. The population of a town grew by 5% in one year. If the population started at 700, by how much did the population grow? What is the new population?

Challenge Problems

13. The population of a small town was 3 000 in 2012.
    (a) If the population in 2013 were 13% more than the population in 2012, what would the 2013 population be?
    (b) If the population in 2014 were 13% less than the population in 2013, show that the 2014 population would not return to 3 000. Explain this apparent paradox.

14. Coffee beans lose 12.5% of their weight during roasting. In order to obtain 252 kg of roasted coffee beans, how many kg of unroasted beans must be used?

15. The number of donations to a charity increased by 25% two years ago and then decreased by 25% last year. The number of donations is now 4500 persons. How many donations were there before the two changes?

Proportions

1. If I can peel 12 apples in 8 minutes, how long will it take me to peel 6 apples at the same rate?

2. (Continuation) How many apples can I peel in 1 minutes?

3. (Continuation) A *proportion* is an equation stating that two ratios are equivalent. Using your answers to the two previous problems, write three equivalent ratios.

4. Ryan took 25 seconds to type the final draft of a 1000-word English paper. How much time should Ryan expect to spend typing the final draft of a 4000-word History paper?

5. (Continuation) The *proportion* $\frac{25}{1000} = \frac{x}{4000}$ is helpful for the previous question. Explain this proportion, and assign units to all four of its members.

6. If I run 100m in 20 seconds, how many metres can I run in 50 seconds at that rate?

7. (Continuation) If I run 100m in 20 seconds, how many metres can I run in 1 second at that rate?

8. (Continuation) If I run 100m in 20 seconds, how many seconds will it take me to run 20m at that rate?

9. (Continuation) If I run 100m in 20 seconds, how many seconds will it take me to run 1 metre at that rate?

10. Claire is taking a trip from Stratford to Paris, and she needs to exchange 500 British pounds for euros. The exchange rate is 1 pound to 1.23 euros. How many euros will she receive in this exchange?

11. When Aviva went to Tanzania, she estimated that she would need about 1.6 million Tanzanian shillings for her trip. The exchange rate at the time was one U.S. dollar to 2143 shillings. How many U.S. dollars did Aviva need to make this exchange?

12. A blueprint of a building gives a scale of 1 inch = 8 feet. If the blueprint shows the building sitting on a rectangle with dimensions 16 inches by 25 inches, what are the actual dimensions of the building? What is the actual area of the building?

SI Units

1. There are 1 000 metres in 1 kilometre. How many metres are in 5 kilometres?

2. How many kilometres are there in 7 000 metres?

3. How many metres are in 3.5 kilometres?

4. There are 100 centimetres in 1 metre. How many centimetres are there in 14.29 metres?

5. How many centimetres are there in 1 kilometre?

6. On a road map of Uganda, the scale is 1 : 1 500 000. The distance on the map from Kampala to Ft. Portal is 17cm. What is the real world distance in km between these two cities?

7. There are 10 millimetres in 1 centimetre. How many millimetres are there in a kilometre?

8. Each side of a square plot of land is 1 kilometre. What are its dimensions in metres? What is its area in m^2?

9. On a map of South Asia, Nepal looks approximately like a rectangle measuring 8.3 cm by 2.0 cm. The map scale is listed as 1 : 9 485 000.
   (a) What are the approximate real world dimensions of Nepal in centimeters?
   (b) What are the approximate real world dimensions of Nepal in kilometers?
   (c) What is the approximate real world area of Nepal in km^2?

10. Given the task of converting 5 km^2 into cm^2, Rehman did the following set of calculations:

$$5km^2 \times \frac{1000m}{1km} \times \frac{1000m}{1km} \times \frac{100cm}{1m} \times \frac{100cm}{1m} = 5 \times 1000 \times 1000 \times 100 \times 100cm^2 = 5 \times 10^{10} cm^2$$

    What do you think of his approach?

11. Convert 23000 mm^3 into m^3.

12. One cubic centimetre is also called one millilitre. One thousand millilitres are in one litre and one thousand litres are in one kilolitre. How many cubic *metres* are in one kilolitre?

Calculating Profit and Loss

1. A group of school children spend E30 buying supplies to make batiks.
   (a) If they sell 5 batiks for E10 each, how much *revenue* will they have? How much *profit* have they made?
   (b) If they sell 3 batiks for E10 each, how much revenue will they have? Will they have a profit? A loss?
   (c) If they sell 3 batiks for E9 each, how much revenue will they have? Will they have a profit? A loss?

2. Mr. Msebenzi buys 480 sweets for E50.00. He repacks the sweets into packets of 24 each.

(a) Would he have a profit if he sold all of the packets for E2? If so, how much is his profit? If not, what would his loss be if he had one?

(b) Would he have a profit if he sold all of the packets for E2.50? If so, how much is his profit? If not, what would his loss be if he had one?

(c) Would he have a profit if he sold all of the packets for E3? If so, how much is his profit? If not, what would his loss be?

(d) Would he have a profit if he sold only 15 packets for E3? If so, how much is his profit? If not, what would his loss be if he had one?

3. The prices at a store rose by 20%. By how much did the price of a E100 shirt rise? What is the new price of the shirt?

4. A store decided to raise the price of everything they had by 23%. If a shirt was originally priced at R100,00, by how much will the price increase? What will the new price of the shirt be?

5. Andrea has heard that prices in her grocery store are rising by 23%. She wants to purchase an item that used to cost E15,00.

   (a) By how many rand did the cost of the item increase?

   (b) How much will the item cost now?

   (c) What percentage of E15,00 does the item now cost?

6. Nadia is buying a $450,00 item and the VAT is currently set at 14%. In this country, the quoted price of the item does *not* include the VAT.

   (a) How much did Nadia pay in taxes?

   (b) How much total did Nadia pay for the item?

   (c) What percentage of the retail price did Nadia pay for the item?

7. Nadia is now buying a P dollar item and the VAT is still set at 14%. Again, in this country, the quoted price of the item does *not* include the VAT. What percentage of P will Nadia pay for the item? How much will Nadia pay for the item? Express your answer in terms of P.

8. Nadia needs to buy a P dollar item in a country with a VAT of r%. Again, in this country, the quoted price of the item does *not* include the VAT. How much will Nadia pay for the item? Express your answer in terms of r and P.

9. Wes bought some school supplies at a store in a country that has a 6.5% VAT. In this country, the quoted price of the item does *not* include the VAT. He bought two blazers priced at R500,00 and three pairs of pants priced at R210,00. How much total did Wes pay?

10. (Continuation) A familiar feature of arithmetic is that *multiplication distributes over addition.* Written in algebraic code, this property looks like $a(b+c) = ab+ac$. Because of this property, there are two equivalent methods that can be used to compute the answer to the previous problem. Explain, using words and complete sentences.

11. Sarah has a coupon for 25% off at the grocery store. Suppose she wants to purchase R750,00 worth of groceries
    (a) How much will Sarah be saving?
    (b) How much will Sarah spend total?
    (c) What percentage of the full-price cost will Sarah pay?

12. Yunus has four items in his shopping basket with the following prices: R401,00, R324,00, R650,00, and R125,00. He also has a coupon for 40% off.
    (a) How much will Yunus be saving?
    (b) How much will Yunus spend total?
    (c) What percentage of the full-price cost will Yunus pay?

13. Carl has four items in his shopping basket with the following prices: R75,25, R69,99, R38,92, and R24,25. He has a coupon for 10% off. What percentage of the full-price will Carl pay for his items? How much will Carl pay total?

14. Akila wants to buy a E100,00 football jersey and has a coupon for r% off. What percentage of the full-price will Akila pay for his jersey? How much will Akila pay? Express your answers in terms of r.

15. Now Akila wants to buy a jersey that costs P rand and he has a coupon for r% off. How much will Akila pay? Express your answers in terms of r and P.

16. Woolworth's had a going-out-of-business sale. The price of a telephone before the sale was E475,00. What was the price of the telephone after a 30% discount? If the sale price of the same telephone had been E285,00, what would the (percentage) discount have been?

17. Last week, Chris bought a DVD for E128,00 while the store was having a 25% off sale. The sale is now over. How much would the same DVD cost today?

18. Last year, the price of an iPod was E2852,00.
    (a) This year the price increased to E3090,00. By what percent did the price increase?
    (b) If the price next year were 5% more than this year's price, what would that price be?
    (c) If the price dropped 5% the year after that, show that the price would not return to E3090,00. Explain this apparent paradox.

19. Corey deposits E3000,00 in a bank that pays 4% annual interest. How much interest does Corey earn in one year? How much money is in Corey's account after one year?

20. Corey deposits E3000,00 in a bank that pays 6% annual interest. How much interest does Corey earn in one year? How much money is in Corey's account after one year?

21. One year after Robin deposits E4000,00 in a savings account that pays $r\%$ annual interest, how much money is in the account? Write an expression using the variable r.

22. One year after Robin deposits P emalangeni in a savings account that pays $r\%$ annual interest, how much money is in the account? Write an expression in terms of the variables P and r. If you can, write your answer using just a single P.

Activities

Mathland Money

These activities can be used to teach basic operations, such as addition, subtraction and multiplication with regrouping, and long division while reinforcing the ideas of place value.

Materials:

- Lots of card paper cut into rectangles to represent the fictional currency of MathLand. The denominations should be M10 000, M1 000, M100, M10 and M1.

Steps for Long Division:

1. Split the students into groups of 3 or 4. Choose one student in each group to serve as the banker.
2. Tell the students that Farah and 6 of her friends were playing one day when they discovered an envelope with nine M1000 bills, eight M100 bills, two M10 bills and eight M1 bills. Have the banker pull out the found money and give it to the remainder of the group.
3. Ask the students the following questions:
   (a) How much total money did Farah and her friends find?
   (b) What do you think would be a fair way to split the money between the 7 people? How much would each person get?
4. In a whole-class discussion, ask the students how much each person should get and how they determined their answer. At least one of the groups will have struck upon the idea of changing the M1000 for M100 and so forth, but if they haven't, you should bring it up.
5. Show the students the long-division for 9828 by 7. Ask the students where each number in the process was found in their money changing activity.
6. Ask the students to complete the following problems by first splitting money fairly and then by writing out the long division:
   (a) $408 \div 4$ (b) $1208 \div 8$ (c) $45\,305 \div 5$ (d) $492\,060 \div 12$
7. Ask the students to use long division to calculate $72\,135 \div 35$.

Extension:

This activity can be extended to teach conversion of common fractions into decimal fractions through long division of the numerator by the denominator. You would need "coins" of the following denominations: M0.10 and M0.01, 10-cent and 1-cent coins, respectively. As above, ask the students how a mother could fairly divide M250 between 8 children, making change in each step. Then show the long division and ask the students where each number in the process can be found in their money-changing activity.

Fraction Cards
This activity can be used in a variety of contexts for teaching fractions, such as equivalent fractions and fraction addition.
Materials:

- Square or graph paper cut into the following:
  - several 12×12 squares and mark them with "1" to represent a whole.
  - a couple of 24×12 rectangles and mark them with "2" to represent two wholes.
  - several 6×12 rectangles and mark them with "$\frac{1}{2}$".
  - several 4×12 rectangles and mark them with "$\frac{1}{3}$".
  - several 3×12 rectangles and mark them with "$\frac{1}{4}$".
  - several 2×12 rectangles and mark them with "$\frac{1}{6}$".
  - several 1×12 rectangles and mark them with "$\frac{1}{12}$".

Steps:

1. Ask the students to use the cutouts to explain why $\frac{2}{4}$ could be said to be the same as $\frac{1}{2}$.
2. Explain the concept of *equivalent fractions* and ask the students to use the cut outs to find other equivalent fractions and list them in their notebook.
3. Discuss as a whole-class any patterns the students have found in how to find equivalent fractions.
4. Ask the students to find an equivalent fraction for $\frac{3}{12}$.
5. Check if the students understand the concept by having themfind an equivalent fraction for $\frac{6}{24}$ even though they do not have any cut outs for 24ths.
6. Ask the students to find an equivalent fraction for the result when $\frac{1}{3}$ is added to $\frac{1}{4}$.
7. Have the students find equivalent fractions with denominators of 12 for $\frac{1}{3}$ and $\frac{1}{4}$ and ask them how this helps with the previous problem.
8. Have the students use the cut outs to solve the following addition problems:
   (a) $\frac{1}{2} + \frac{1}{4}$ (b) $\frac{2}{3} + \frac{1}{6}$ (c) $\frac{4}{3} + \frac{1}{6}$ (d) $\frac{3}{4} + \frac{5}{6}$

Counting Steps
"A journey of a thousand kilometres begins with a single step." This saying is the basis for the following activity during which students use unit conversion and simple proportions to estimate how many of their steps are in a thousand kilometres.
Materials:

- A ruler with centimetre markings

Steps:

1. Tell the students the saying and ask them to estimate how many steps they would each need to take to walk 1000 kilometres using the ruler with centimetre markings.

2. Let them struggle and exercise their problem solving skills. If they get stuck, here are some questions to ask to prompt them in the right direction:

   (a) How many centimetres are in a single one of your steps?

   (b) How many metres are in a single step?

   (c) How many steps are in a single metre? (They can use a simple proportion here)

   (d) How many steps are in a kilometre?

   (e) How many steps are in 1 000 kilometres?

   Answers will vary but should all be around the same.

Estimating Capacity

In this activity, students will determine approximately how many candies various boxes can hold.

Materials:

- A beaker with a known capacity (500ml, 1l, etc.)
- Smarties or similar candy
- Ruler with mm markings
- Boxes of various sizes such as cereal boxes, a jewelry box, and so forth

Steps:

1. Fill the beaker with the candy.

2. Pour the candy out and have the students count how many pieces fit into the beaker. This provides a density.

3. Give the students the dimensions of the boxes in **meters**.

4. Have the students determine the volume of the boxes in $\mathbf{cm}^3$.

5. Ask the students to determine approximately how many pieces of candy will fit in the box. They will need to use the idea that $1\text{cm}^3 = 1\text{ml}$.

6. (*Optional*) Check their estimates by filling the boxes with candy and determining exactly how many pieces of candy fit in the box. Ask them to discuss why their answers were different from the actual amount.

Juniper Green
This game was described by Ian Stewart in his Mathematical Recreations column in the March 1997 issue of Scientific American, page 118. Juniper Green is the name of the school where it was first played. It was originally designed to teach multiplication and division. Students gain familiarity with multiples and factors through this game while engaging their problem solving skills as they attempt to find a winning strategy.
Materials:

- Card paper cut into rectangles with a number from 1 to 100.
- OR A large 10 x 10 chart with the numbers 1 through 100 written and counters or markers to cover each number as it is chose.

The Rules:

1. Two players take turns removing a card (or covering a number). Cards removed are not replaced and not used again.
2. The first card chosen must be even.
3. Apart from the first move each card taken must be an exact multiple of the previous card or an exact divisor of the previous card.
4. The first card taken must be an even number.
5. The first player unable to take a card loses.

Extension:
Call JG-n a game of Juniper Green with cards 1 through n. For which values of n from 2 through 10 in JG-n does the first player have a winning strategy?

Prelude: Workshop focus considerations brad.uy@gmail.com

6-9 Constructions and Probability

Constructions

I. Inventory
   A) List of necessary constructions for grade levels
      1. What I found for grades 6 and 7 and 11 and 12? Grade 6, objectives 2a-c: Use compasses and ruler to construct **triangles when given the lengths of the three sides. Construct a triangle given any two sides and an angle between them; Construct a triangle given any side and two angles.** Grade 7, objectives 1 and 2, **Construct a 60^0 angle** using a pair of compass and ruler only; **Bisect angles** by folding and by using a pair of compass and ruler. Grade 11 and 12, adds **perpendicular bisector**

II. What we cover in this outline
   A) Copying
      1. Construct a congruent segment; a congruent angle (see diagram below)
      2. Construct a congruent triangle
   B) Bisect
      1. Create a perpendicular bisector of a segment (which also finds the _____ of the segment) (see diagram below)
      2. Bisect any angle
   C) How do we get a 60-degree angle?
   D) Perpendicular to a line at a point on the line; perpendicular to a line from a point not on the line
      1. the key here is establishing a new start--going back a step--stabilize and orient!
   E) Drawing a parallel line to a given line
      1. hint: relates to copying an angle--but the orientation is tricky at times!!
   F) EXTRA PRACTICE
      1. Constructing a hexagon….
      2. Constructing an octagon….
      3. Constructing a parallelogram; or rhombus.

IV. Resources and Materials
   A) Compass, straightedge, and pencil--no protractor of course!
   B) https://www.mathsisfun.com/geometry/constructions.html
      1. this website has it all; amazing animations
      2. your workbooks have excellent directions, especially for exotic shapes

V. What is the main function of a compass? I.e., what can it do fundamentally? What is its purpose?
   A) Understanding the tool…

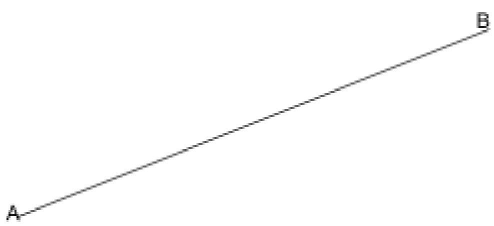
VI. Proving triangles congruent
   A) What is allowed?
      1. Theorems: SSS, SAS, ASA, AAS, and HL (the last only for right triangles)
   B) What's not?
      2. ASS, bad word!!
   C) Helps provide the logic to justify constructions, via CPCTC, corresponding parts of congruent triangles are congruent.

VII. Dos and Don'ts when performing constructions
   A) Dos
      1. Remember that lines or line segments are defined by two points. If we need to form lines, the question that lies at the heart of constructions is how do we **form points**?

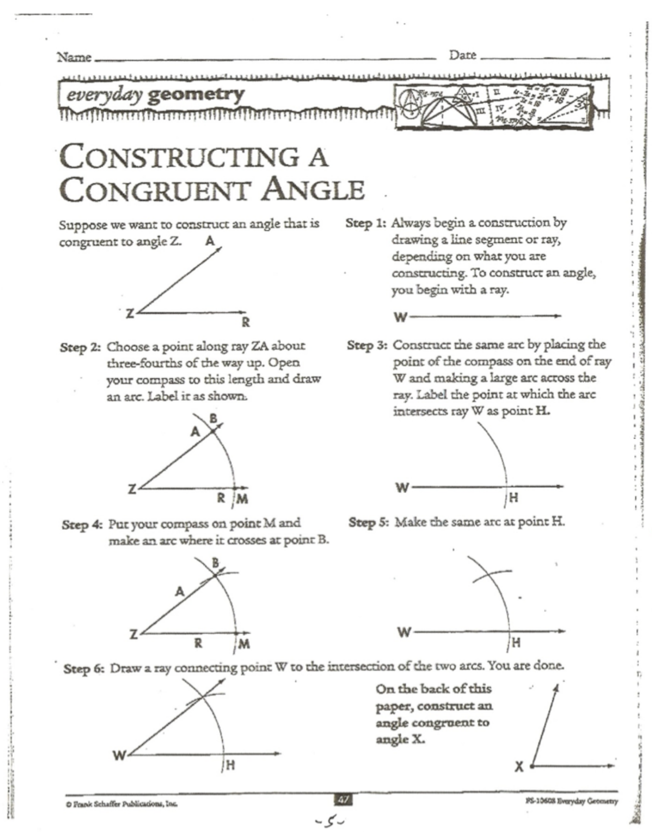
a) they could be given (as a point outside a line, or as the intersection of two rays as in a vertex of an angle)
b) create them
c) defined at the intersections of
   1. 2 lines
   2. A line and an arc
   3. 2 arcs
d) most commonly, we create points by numbers 2 and 3 above.

2. Form a gameplan before beginning the construction. Sketch what the final drawing will look like and ideas on how to achieve it.

B) Don’ts
1. Do not use a protractor or ruler to measure angles or lengths
2. Make arcs that are too short
3. Start with lines that are too short



1.construct a congruent segment above



Name ______ Date ______

everyday geometry

CONSTRUCTING A CONGRUENT ANGLE

Suppose we want to construct an angle that is congruent to angle Z.

Step 1: Always begin a construction by drawing a line segment or ray, depending on what you are constructing. To construct an angle, you begin with a ray.

Step 2: Choose a point along ray ZA about three-fourths of the way up. Open your compass to this length and draw an arc. Label it as shown.

Step 3: Construct the same arc by placing the point of the compass on the end of ray W and making a large arc across the ray. Label the point at which the arc intersects ray W as point H.

Step 4: Put your compass on point M and make an arc where it crosses at point B.

Step 5: Make the same arc at point H.

Step 6: Draw a ray connecting point W to the intersection of the two arcs. You are done.

On the back of this paper, construct an angle congruent to angle X.

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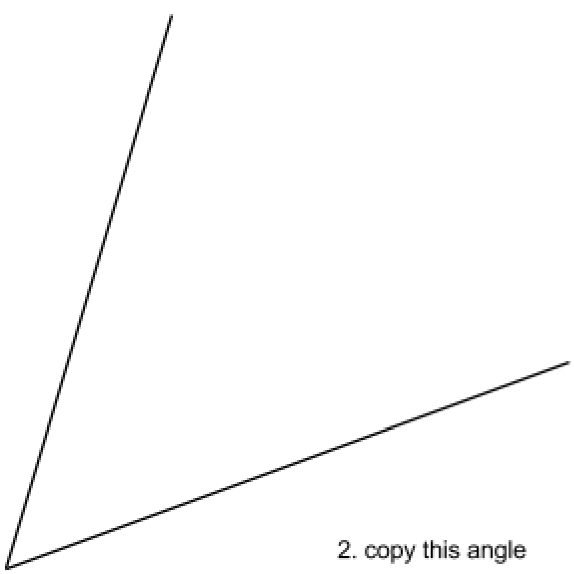
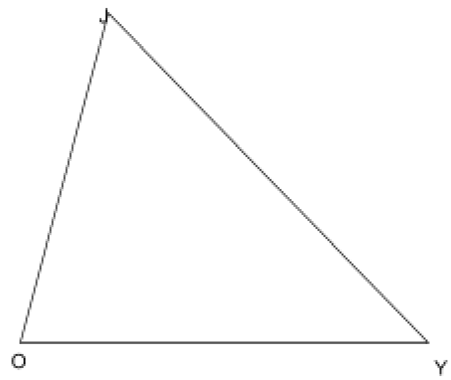


2. copy this angle



3. construct a triangle congruent to triangle JOY

4. constructing a perpendicular bisector

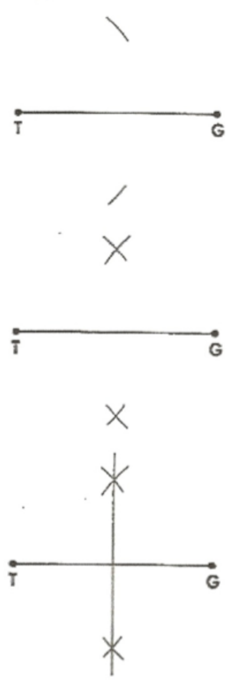
Name ______________________ Date __________

everyday geometry

Constructing a Perpendicular Bisector

Perpendicular means that two lines or segments meet at a right angle, like an upside down letter T. When something like a segment or angle is bisected, it is divided in half. In this activity, you will construct the perpendicular bisector of a line segment.

Suppose we wanted to bisect the segment TG.



Step 1: Open your compass and place it on point T.

Step 2: Extend the compass to any point greater than half the length of TG.

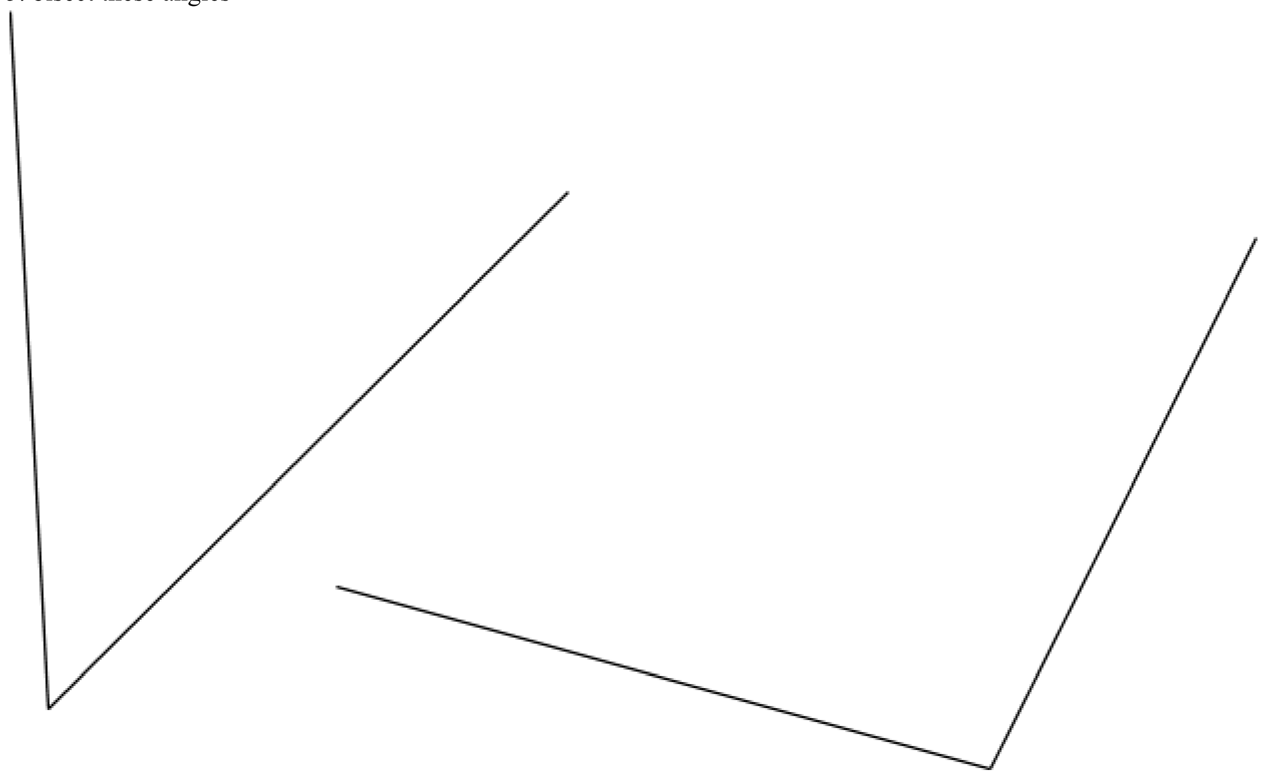
Step 3: Construct an arc below and above TG.

Step 4: Put the compass on point G and construct the same arcs. Make sure you intersect the arcs you drew in step 3.

Step 5: Draw the bisector where the arcs intersect.

Construct the perpendicular bisector of segment AH below.

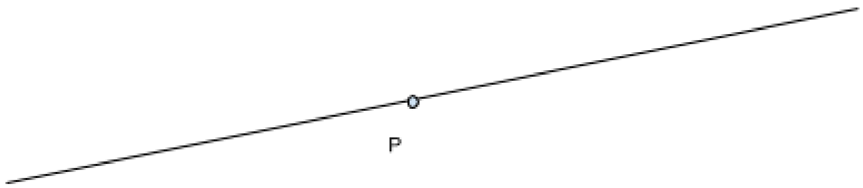
5. bisect these angles

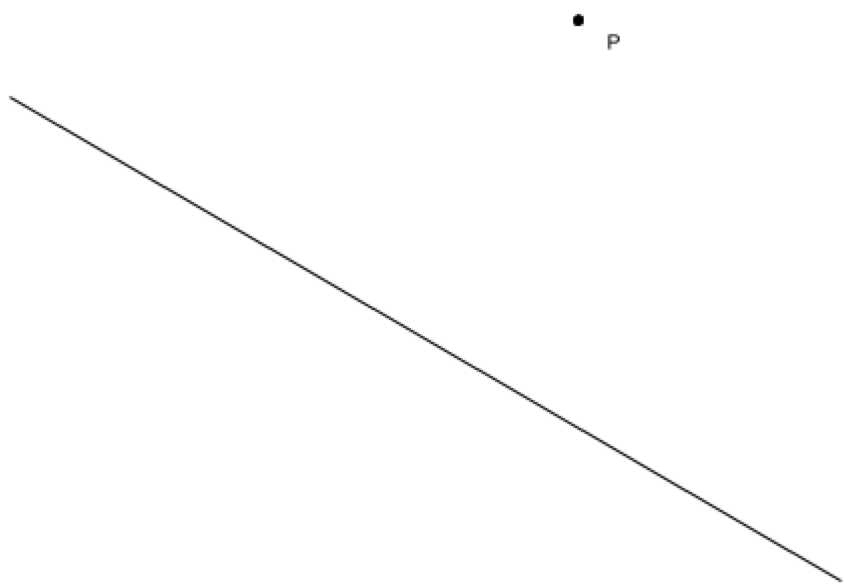


6. now that you know how to do that….below, try to construct a 60 degree angle…and then a 30 degree angle.

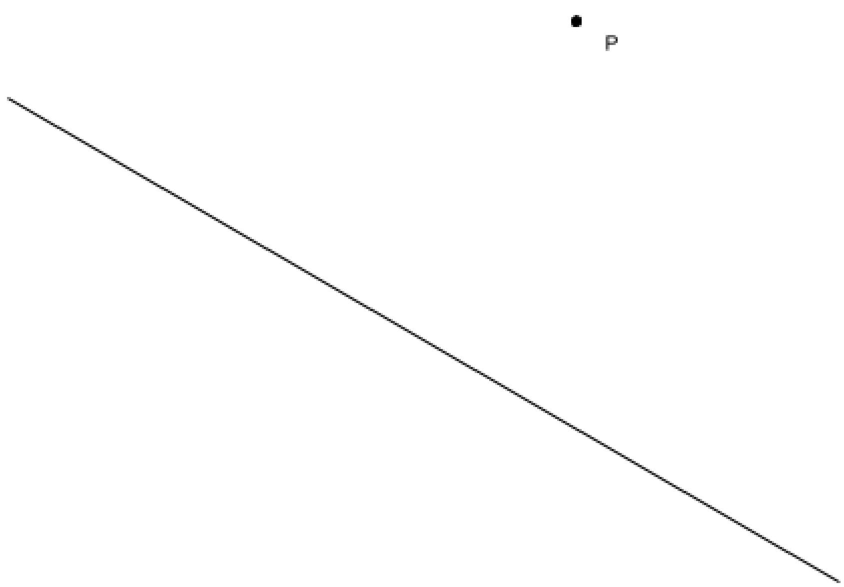
P.S. how do we get a 45 degree angle?

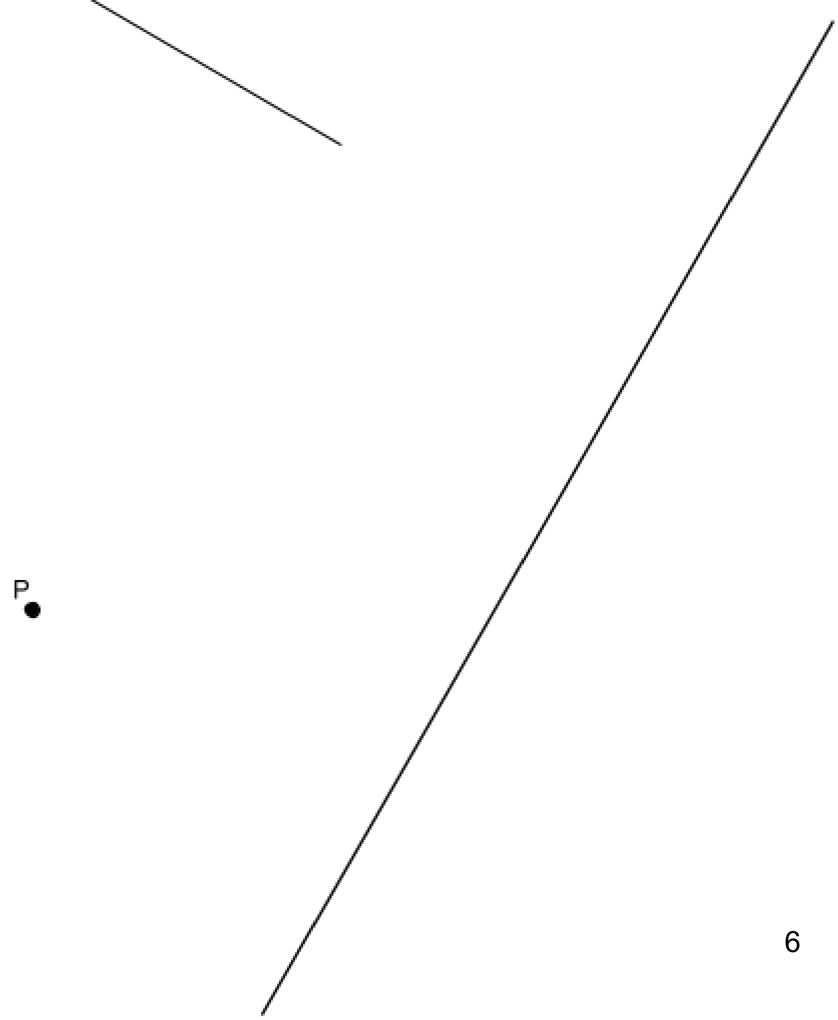
draw a perpendicular to the line through point P





draw a line parallel to the given line through point P





try constructing a hexagon, an octagon, or a parallelogram, or a rhombus!

Probability

Probability language

0. Birthday test

I. Brainstorming and exchange

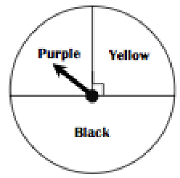
II. Introduction
    A) constructing a sample space
        1. using tables and tree diagrams to help
    B) Definition of probability
        1. two coins; two dice and their sums; graduate to three births
        2. evaluating respective probabilities
    C) Probability is a number--sliding scale
        1. Placing words, impossible, even chance, certain, somewhat likely, somewhat unlikely, “possible”

III. Analytic aids
    A) Tree diagrams
        1. following through branches with multiplication
            a) adding outcomes in the end
                i. application of addition rule

P1 What kinds of questions can we ask to get students thinking about measuring likelihood? What scale can we do it in? Examples: Probability of dead or alive; probability of being a male or female. Probability of being dead and alive at the same time; probability of flipping a coin and a bunny hop out of it; probability of… Offer us some less outlandish questions, though illustrative of definitions of probability.

P2 Will this work? Other ideas? Blends probability and data handling but we could poll those with families of 2 or greater. Record the frequency of BB, BG, GB, and GG and compare that to our theoretical model. What other projects can we do to explore relative frequency v probability concepts?

7.2 The spinner below is spun twice in succession.

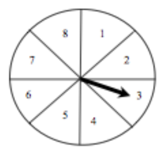


7.2.1 What is the probability that the arrow will point to yellow after the first spin and to black after the second spin?

P3

QUESTION 8

8. If the spinner below is rotated, determine the probability that the arrow will point to:



8.1 A 1 or a 3?
8.2 A number greater than 2?
8.3 A factor of 8.

Debrief

Possible suggested lab--demonstration type lessons (activities all detailed in supplement)

1. Introduction. Sliding scale probability intuitive sense of probability as a sliding scale--ask them probability that they'll eat something green tonight;encounter a bicycle on the way home; get sent to the principal later today; get pooped on by a bird; etc. They can even imagine their own events and trade questions with each other.

2. Game. 2 coin toss and have three parts of the room; 1 representing the HH team. the other the TT team and the other representing a tail and a head. Tally--first to 20 wins. QUESTION: does the procedure of coin toss affect outcome and results? What does this tell us about the nature of these two events?
----
transition to another analytic aids….Venn diagrams

8 students take Physics and 4 students take Chemistry, but there are only 10 students we're considering who are enrolled.... How is this possible?

or… There are 20 people who responded to the survey. 13 people like cats. 4 people like both cats and dogs. How many people like dogs? How many like only dogs?

----
http://www.gscdn.org/library/cms/24/25324.pdf?_ga=1.103851395.226993679.1463690423
http://www.gscdn.org/library/cms/22/24922.pdf?_ga=1.175702433.226993679.1463690423http://www.commoncoresheets.com/Math/Probability/Basic%20Probability/English/3.pdf
http://www.mathworksheets4kids.com/probability/balls-container.pdf

Probability scale 0 to 1

Look at this probability line.

Impossible = 0
Poor chance = 0.25
Fair = 0.5
Good chance = 0.75
Certain = 1

Write each letter in the correct place on the probability line.

a. It will be daylight in New Orleans at midnight.
b. The sun will come up tomorrow.
c. If I toss a coin it will come down heads.

a (0) — 0.25 — c (0.5) — 0.75 — b (1)

---

0 — 0.25 — 0.5 — 0.75 — 1

Write each letter in the correct place on the probability line.

a. If I cut a pack of cards I will get a red card.

b. If I cut a pack of cards I will get a diamond.

c. If I cut a pack of cards I will get a diamond, a spade, or a club.

d. If I cut a pack of cards I will get a diamond, a spade, a club, or a heart.

e. If I cut a pack of cards it will be a 15.

---

0 — 0.25 — 0.5 — 0.75 — 1

Write each letter in the correct place on the probability line.

a. Next week, Wednesday will be the day after Tuesday.

b. There will be 33 days in February next year.

c. It will snow in Miami in May.

d. It will snow in Chicago in January.

e. The next person to knock on the door will be a woman.

Probability

Mark each event on the probability line.

impossible (c) — poor chance — even chance (b) — good chance — certain (a)

a) It will get dark tonight.
b) When I toss a coin, it will land showing heads.
c) Abraham Lincoln will come for lunch.

Mark each event on the probability line.

impossible — poor chance — even chance — good chance — certain

a) Snow will fall in August.
b) The sun will come up tomorrow.
c) A new baby will be a boy.
d) A dog will speak English.
e) I will watch some television tonight.

Mark each event on the probability line.

impossible — poor chance — even chance — good chance — certain

a) I will roll a 6 on a number cube.
b) I will not roll a 6 on a number cube.
c) I will roll a number between 1 and 6 on a number cu
d) I will roll a 7 on a number cube.
e) I will roll a 1, a 2, or a 3 on a number cube.

Mark each event on the probability line.

impossible — poor chance — even chance — good chance — certain

a) I will drink something today.
b) If I drop my book, it will fall face down.
c) The next book I read will have exactly 100 pages.
d) It will rain orange juice tomorrow.
e) I will see a white car today.

Basic Probability

Name:

Use each diagram to solve the problems.

1) How many pieces are there total in the spinner?

2) If you spun the spinner 1 time, what is the probability it would land on a gray piece?

3) If you spun the spinner 1 time, what is the probability it would land on a black piece?

4) If you spun the spinner 1 time, what is the probability it would land on a white piece?

5) If you spun the spinner 1 time, what is the probability of landing on either a gray piece or a white piece?

6) If you were to roll the dice one time what is the probability it will land on a 4?

7) If you were to roll the dice one time what is the probability it will NOT land on a 4?

8) If you were to roll the dice one time, what is the probability of it landing on an odd number?

9) How many shapes are there total in the array?

10) If you were to select 1 shape at random from the array, what is the probability it will be a diamond?

11) If you were to select 1 shape at random from the array, what shape do you have the greatest probability of selecting?

12) Which shape has a 35% chance (7 out of 20) of being selected?

Answers

1. 8
2. 2 out of 8
3. 3 out of 8
4. 3 out of 8
5. 5 out of 8
6. 1 out of 6
7. 5 out of 6
8. 3 out of 6
9. 20
10. 8 out of 20
11. diamond
12. heart

Balls in a container

	Work Space
There are 5 white balls, 8 red balls, 7 yellow balls and 4 green balls in a container. A ball is chosen at random. What is the probability of choosing red? Answer: What is the probability of choosing green? Answer: What is the probability of choosing either red or white? Answer: What is the probability of choosing neither white nor green? Answer: What is the probability of choosing other than yellow? Answer: What is the probability of choosing black? Answer:	

http://www.math-aids.com/Probability/
http://www.math-aids.com/cgi/pdf_viewer_8.cgi?script_name=probability_definition.pl&x=144&y=29

Definitions for Probability

Probability

Probability is the likelihood of the occurance of an event. The probability of event A is written P(A). Probabilities are always numbers between 0 and 1, inclusive.

The four basic rules of probability:

1) For any event A, $0 \le P(A) \le 1$.
2) P(impossible event) = 0.
   Also written P(empty set) = 0 or $P(\emptyset) = 0$.
3) P(sure event) = 1.
   Also written P(S) = 1, where S is the sample set.
4) P(not A) = 1 - P(A).
   Also written P(complement of A) = 1 - P(A) or
   $P(A^c) = 1 - P(A)$ or $P(\bar{A}) = 1 - P(A)$.

NOTE: Multiplication rule is what we apply in cases of independent and dependent events.

A) Relevant as we're talking about a sequence of separate events in trials
   1. easy if they are independent
   2. adjust accordingly if they are dependent

B) underscores that "and" in context of probability implies what operation

----

Probability : Independent Events

Independent Events

Independent Events are not affected by previous Events.

A coin does not "know" it came up heads before ...

... each toss of a coin is a perfect isolated event.

When rolling a pair of dice, one die does not affect the outcome of the other die ...

... each die is an isolated event.

$$\text{Probability of an event happening} = \frac{\text{Number of ways it can happen}}{\text{Total number of outcomes}}$$

Probability of getting a "Head" when tossing a coin?

$$P(\text{Head}) = \frac{\text{"Head"}}{\text{"Head and Tail"}} = \frac{1}{2}$$

Probability of rolling a "4" on a die?

$$P(4) = \frac{\text{"4"}}{\text{"1","2","3","4","5","6"}} = \frac{1}{6}$$

Probability : Independent Events

Two or More Events

You can calculate the probability of two or more Events by multipling the individual probabilities.

So, for Independent Events:

P(A and B) = P(A) x P(B)

Example: Probability of 3 Heads in a Row

For each toss of a coin a "Head" has a probability of 0.5 :

0.5

0.5 x 0.5 = 0.25

0.5 x 0.5 x 0.5 = 0.125

So the Probability of getting three Heads in a Row is 0.125.

WEEK 4

THE SET OF REAL NUMBERS

- Real Numbers
  - Rational Numbers
    - Integers
      - Whole Numbers
        - Natural Numbers
    - Non-Integers
      - Common Fractions
      - Decimal Fractions
  - Irrational Numbers

All real numbers are either positive or negative, except zero which is neither.

All numbers in the set of integers, {…-3, -2, -1, 0, 1, 2, 3...} are either odd or even. Zero is an even number.

The set of whole numbers can be written {0, 1, 2, 3...}

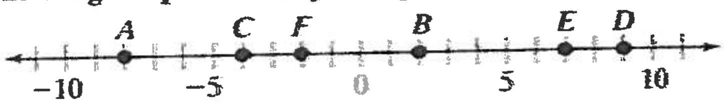
The set of natural numbers, {1, 2, 3, 4...} is the set of positive integers and is also referred to as the set of counting numbers.

The set of irrational numbers includes numbers such as $\sqrt{2}$, $\sqrt[3]{17}$, π, e and any non-terminating, non-repeating decimal.

The set of common fractions includes numbers such as 3/7, -14/3, $3\frac{2}{5}$ etc. A fraction whose numerator is larger than its denominator is sometimes referred to as an "improper" fraction (a poor choice of words!).

The set of decimal fractions (usually simply referred to as decimals) contains any terminating decimal (i.e. 3.75, 0.0027) or repeating decimal (i.e. 4.767676..., $3.\overline{5}$).

Name the integer represented by each point on the number line.



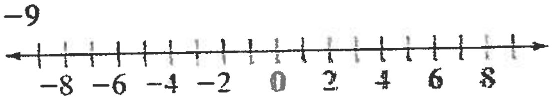
1. A ______ 2. B ______ 3. C ______ 4. D ______ 5. E ______ 6. F ______

Compare. Use <, >, or =.

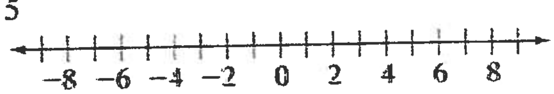
7. $-8 \square 8$
8. $4 \square -4$
9. $-5 \square 1$
10. $-8 \square 0$
11. $-6 \square -2$
12. $-1 \square -3$
13. $-4 \square 0$
14. $-3 \square 2$

Graph each integer and its opposite on the number line.

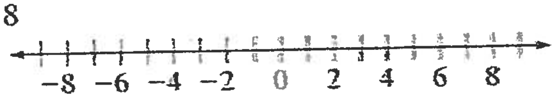
15. -9



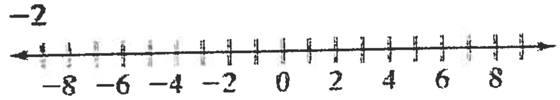
16. 5



17. 8



18. -2



Find the opposite of each number. You may find a number line helpful.

19. 2 ______
20. -3 ______
21. -38 ______
22. $(-2 + 2)$ ______
23. -44 ______
24. $(5 + 2)$ ______
25. -16 ______
26. $(7 - 3)$ ______

Write an integer to represent each situation.

27. a gain of 5 yards ______
28. a debt of \$5 ______
29. a temperature of 100°F ______
30. 135 feet below sea level ______

WEEK 4

NUMBERS, OPERATIONS, & RELATIONSHIPS

PRIMES AND COMPOSITES

A prime number is an integer greater than 1, whose only factors are itself and 1. All other integers greater than 1 are composites (or composite numbers).

In a bookstore, a man found a book with the title *A Complete List of Even Primes*. Describe the pages in this book.

Below is a list of natural numbers. If the number is prime, so indicate; if the number is composite, write the number as the product of its prime factors. The first one is done as an example.

$24 = 2^3 \times 3$ ____________________

31 = ____________________

49 = ____________________

242 = ____________________

76 = ____________________

169 = ____________________

91 = ____________________

93 = ____________________

256 = ____________________

1 = (trick question) ____________________

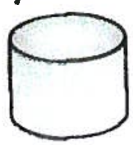
Tests for Divisibility

(To determine whether a number is a factor of a given number)

A number is divisible by	If
2	the last digit of the given number is an even number.
3	the sum of the digits of the given number is itself divisible by 3.
4	the number determined by the last two digits of the given number is itself divisible by 4.
5	the last digit of the given number is 0 or 5.
6	the given number is divisible by both 2 and 3 (use the above tests).
8	the number determined by the last three digits of the given number is itself divisible by 8.
9	the sum of the digits of the given number is itself divisible by 9.
10	the last digit of the given number is 0.
12	the given number is divisible by both 3 and 4 (use the above tests).

WEEK 4

1) cylinder

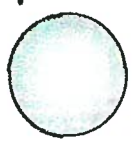


Faces: _____

Edges: _____

Vertices: _____

2) sphere



Faces: _____

Edges: _____

Vertices: _____

3) retangular prism

Faces: _____

Edges: _____

Vertices: _____

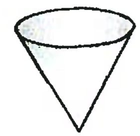
4) triangular prism

Faces: _____

Edges: _____

Vertices: _____

5) cone

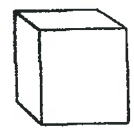


Faces: _____

Edges: _____

Vertices: _____

6) cube



Faces: _____

Edges: _____

Vertices: _____

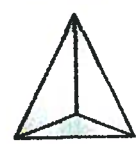
7) square pyramid

Faces: _____

Edges: _____

Vertices: _____

8) triangular pyramid

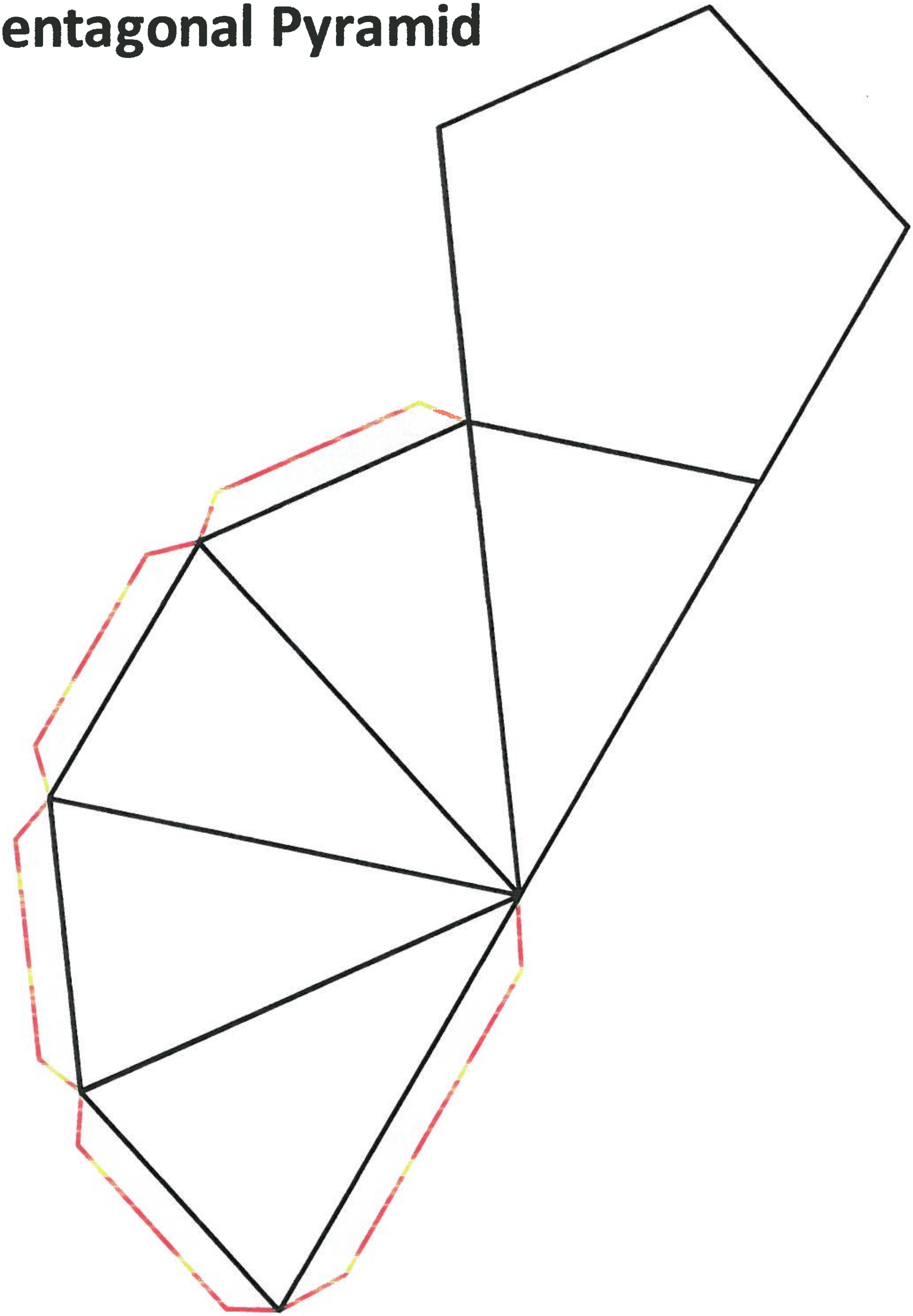


Faces: _____

Edges: _____

Vertices: _____

Pentagonal Pyramid



WEEK 4

Who Am I?

Directions: Use the clues below to determine the name of the 3-D shape each is describing. Be specific with your answers. For example, do not just write pyramid but instead triangular pyramid.

1.) My top and base are both triangles. Who am I?

2.) I have no vertices and my top and base are the same shape. Who am I?

3.) I have only one vertex. Who am I?

4.) All my faces are exactly the same size and shape. Who am I?

5.) I have 6 faces, 12 edges, and 8 vertices. Who am I?

6.) Two of my faces are square and the other 4 are rectangular. Who am I?

7.) I have no vertices or edges. Who am I?

8.) My top comes to a point and my base is a rectangle. Who am I?

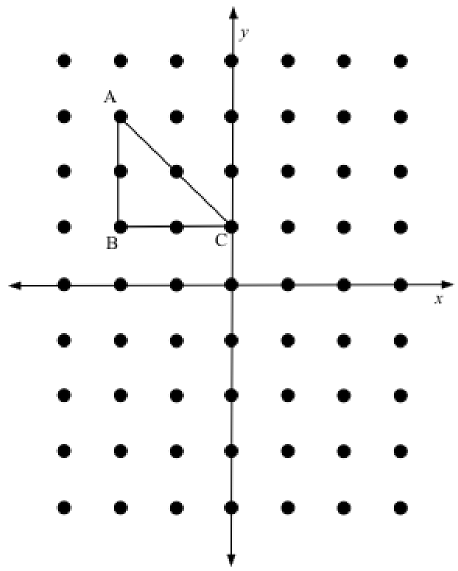
9.) I have 5 faces, 9 edges, and 6 vertices. Who am I?

10.) I have 4 faces and they are all the same shape. Who am I?

Geometry Transformations

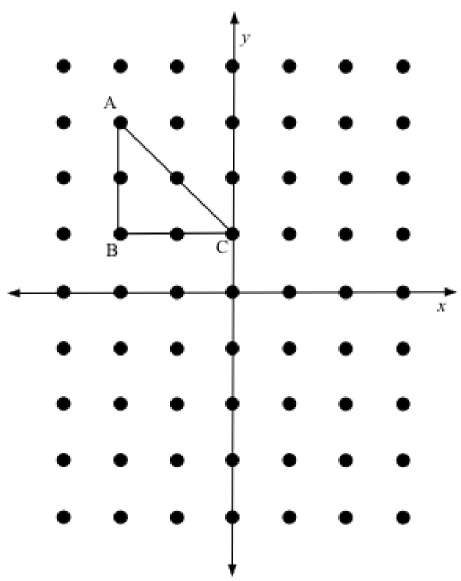
A. **Translation** a given distance in a given direction: A figure is moved up, down, right or left a given distance. This is also called a **glide**. The image is A'B'C.

1. Translate triangle ABC 3 units right and label the image.
2. Translate triangle ABC 4 units down and label the image.



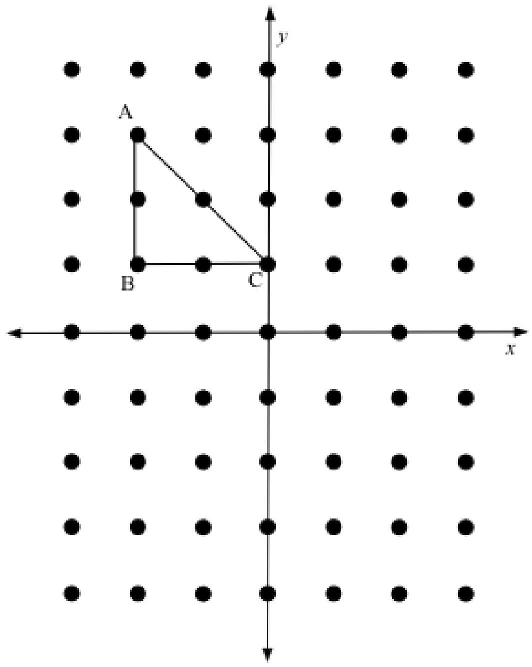
B. **Reflection** over a given line: A figure is reflected over a line. This is also called a **flip**.

1. Reflect triangle ABC over the *y*-axis and label the image.
2. Reflect triangle ABC over the line *y* = 1 and label the image.



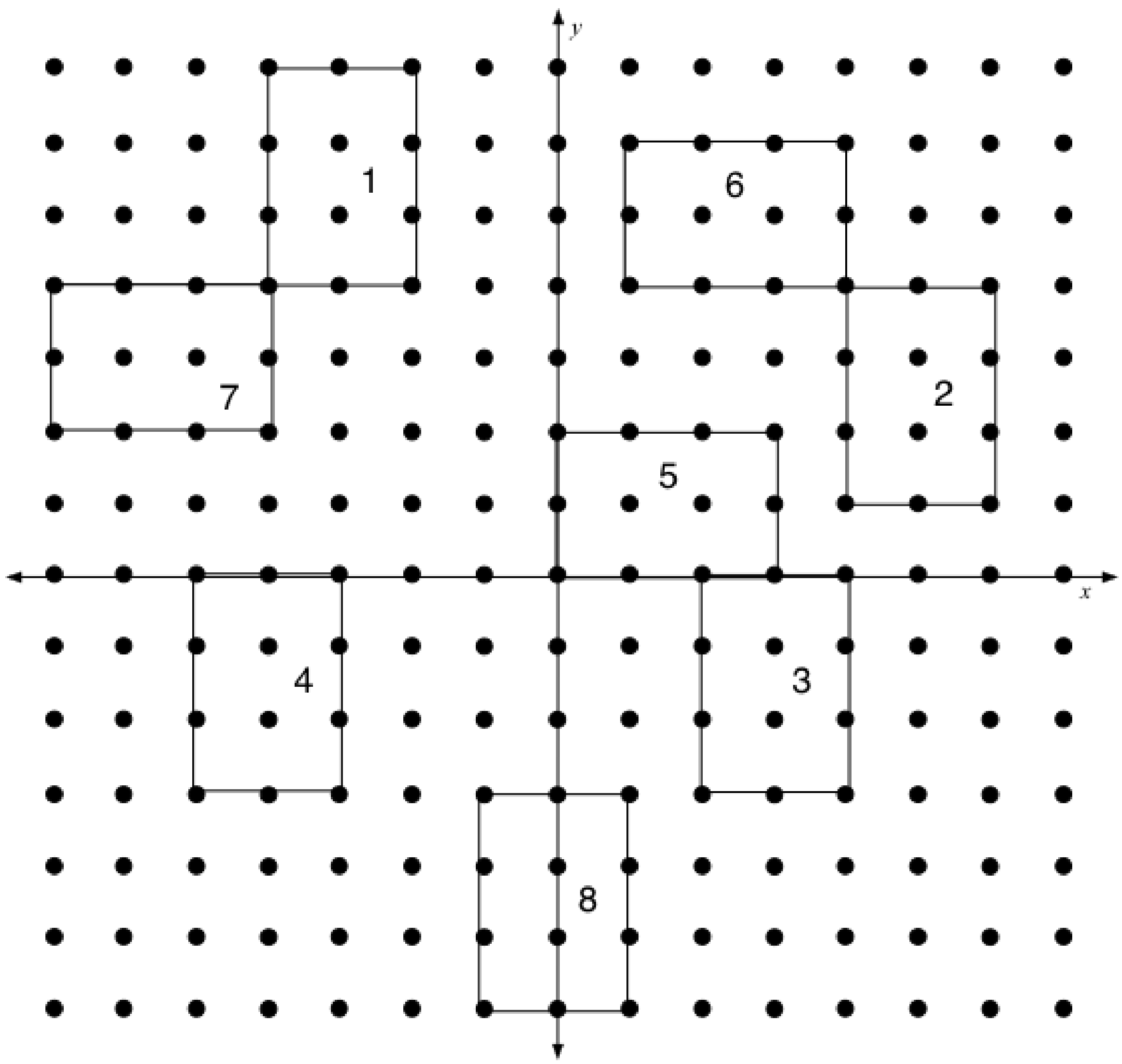
C. **Rotation** About a Given Point: A figure is rotated, or turned, around a given point, a given number of degrees, clockwise or anti-clockwise.

1. Rotate triangle ABC 90˚ clockwise about Point C and label the image.
2. Rotate triangle ABC 180˚ clockwise about the origin and label the image.



Week 4

Follow the instructions and then write the number of the rectangle that matches the figure you moved.



1. Reflect Figure **1** over the *y*-axis. Translate it 3 units down and rotate it 90˚ anticlockwise about (3;1).
2. Translate Figure **2** 1 unit down. Reflect it over the *x*-axis and reflect it over the line $x = 4$
3. Reflect Figure **3** over the *y*-axis. Rotate it 90˚ clockwise around (-2;0) and glide it 5 units right.
4. rotate Figure **4** 90˚ clockwise around (-3;0). Reflect it over the line $y = 2$ and translate it 1 unit left.
5. Translate Figure **5** 5 units left. Rotate it 90˚ clockwise around (-2;2) and glide it up 2 units.
6. Rotate Figure **6** 90˚ clockwise round (-4;4) and reflect it over the line $x = -4$.
7. Rotate Figure **7** 90˚ clockwise around (- 4;4) and reflect it over the line $x = -4$.
8. Reflect Figure **8** over the x-axis. Translate it 4 units left and reflect it over the line $y = 1.5$.

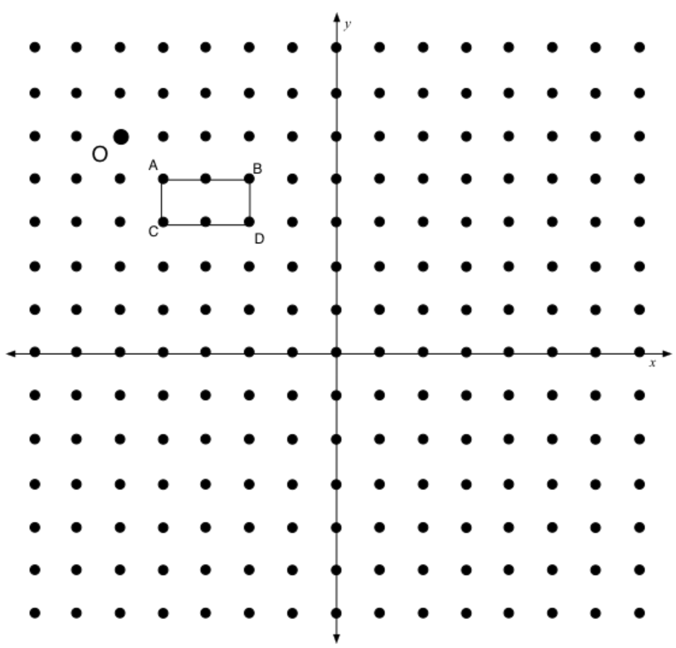
Size Transformations

Dilation: A figure is dilated about a center point according to a scale factor. A scale factor of 1 or -1 will not change the size of the figure.
If the scale factor is larger than 1, the image is larger than the original and this is an **enlargement**;
if the scale factor is less than 1(and positive), the image is smaller than the original and this is a **reduction**.

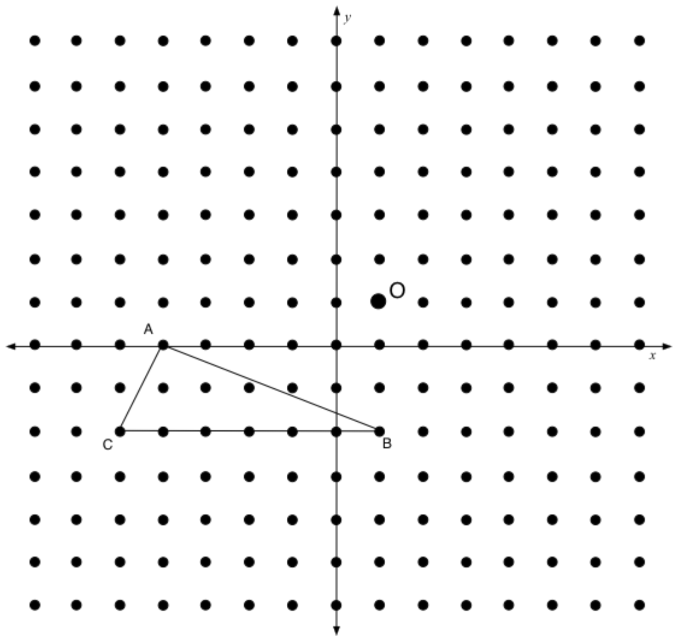
$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}} = \frac{\text{distance of image from center of dilation}}{\text{distance of object from center of dilation}}$$

For every point P:
Step 1: Measure OP.
Step 2: Extend the line OP to the point P' such that OP' = r(OP). where r is the scale factor.
Step 3: Join the points to form the image.

A. Enlarge rectangle ABCD with O as the center and a scale factor of 3. Label the image.

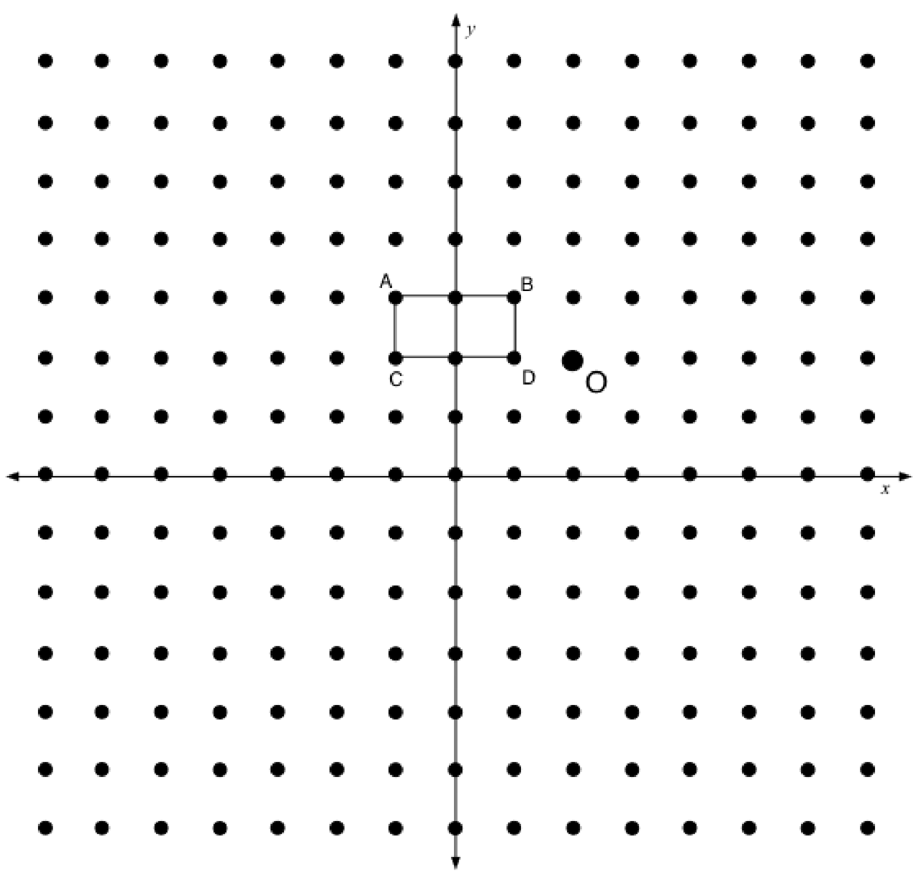


B. Reduce triangle ABC with O as the center and a scale factor of $\frac{1}{3}$.



If the scale factor of a dilation is negative then the image will be on the opposite side of the center of dilation compared with the object.

C. Enlarge rectangle ABCD with O as the center and a scale factor of - 2.



Linear Equations

Knotted Rope

How does the length of a piece of rope change as knots are tied in the rope?

Collect, organize, and represent data as you do the following:

1. Begin with a piece of rope about a metre in length. Measure and record the length of the rope to the nearest cm.

2. Tie one simple overhand knot in the rope (not too tight as you will need to undo all the knots at the end of the experiment). Measure and record the new length of the rope.

3. Tie a second, third, fourth and fifth knot in the rope, and measure and record the new length after each knot is tied. You may want to tie even more knots.

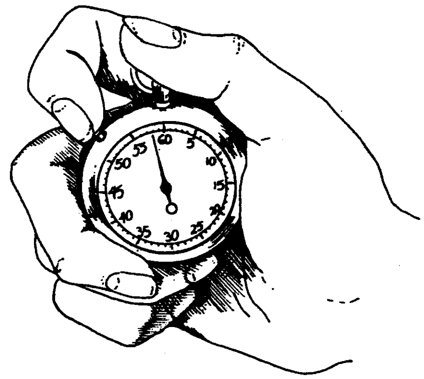
4. Graph the data. Be sure to label the axes.

5. What conclusions can you make from the graph?

6. What is the rate of change in the length of the rope?

7. Write an equation that represents the graph.

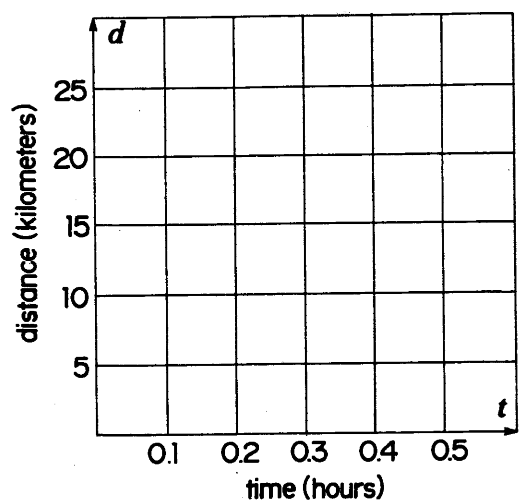
Slope as speed



A navigator in a sports car rally reset her chronometer (a special instrument for measuring time) as the car passed checkpoint C. The table below gives the times and distances traveled to the next five checkpoints. Assume the car speed was constant over this portion of the course.

1. Graph the points and then connect them with line segments.

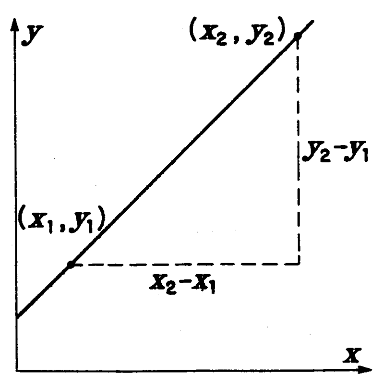
Checkpoint	Time (hours)	Distance (kilometers)
C	0.0	0
D	0.1	5
E	0.2	10
F	0.3	15
G	0.4	20
H	0.5	25



2. At what speed was the car traveling as it passed checkpoint D?____________
   Checkpoint G?____________

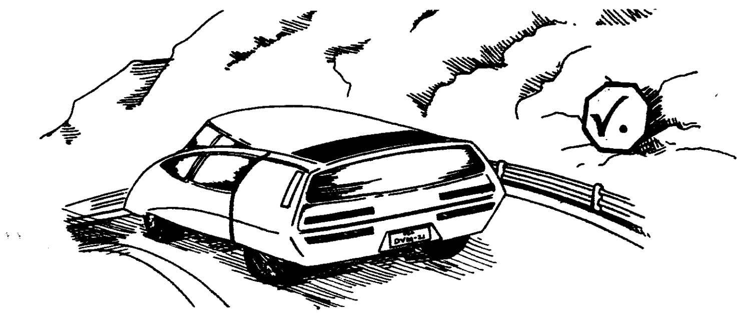
Recall that for any two points (x_1, y_1) and (x_2, y_2) on a nonvertical line, the *slope* of the line is

$$\frac{\text{vertical change}}{\text{horizontal change}} = \frac{y_2 - y_1}{x_2 - x_1}.$$



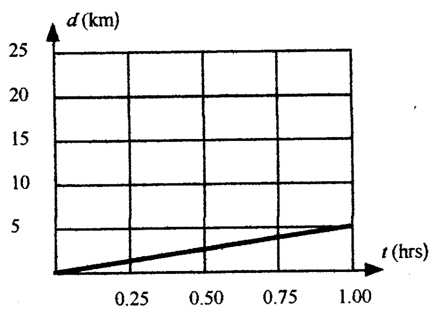
3. *a.* On the graph for exercise 1, label each point with the letter of the checkpoint to which it corresponds.
   b. The slope of the line segment joining points E and F is ________ km/h.
   c. The slope of the line segment joining points D and G is ________ km/h.
4. How do your answers for 3*b* and 3*c* compare with your answers to question 2?

   ____
5. The slope of a distance-time graph is the ____________________.

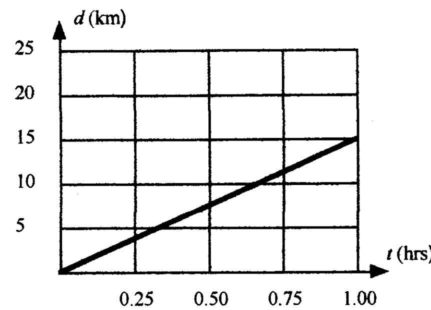


6. Use the slope to find the car speed for each of the following distance-time graphs.

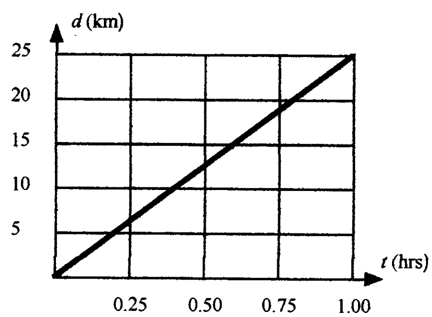
a.



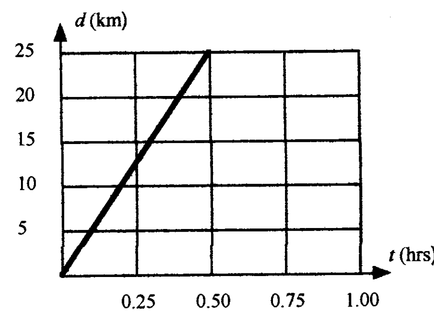
b.



c.



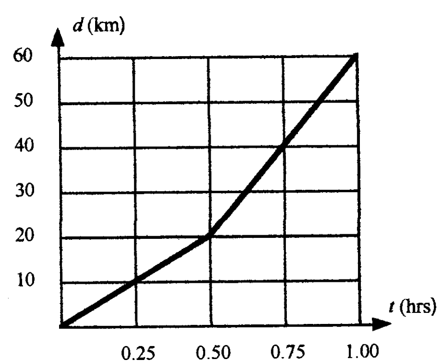
d.



7. Study this distance-time graph and answer the questions that follow.
   a. How fast was the car going during the first 0.5 hours? ____________
   b. How fast was the car going during the second 0.5 hours? ____________
   c. What was the average speed of the car during the 1-hour period?

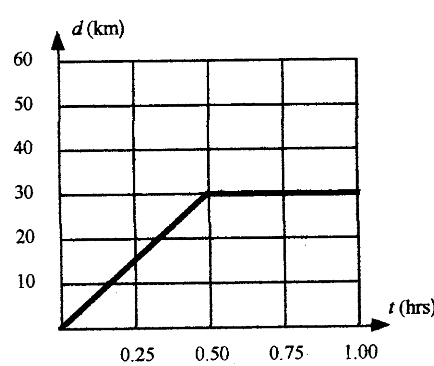
   ____

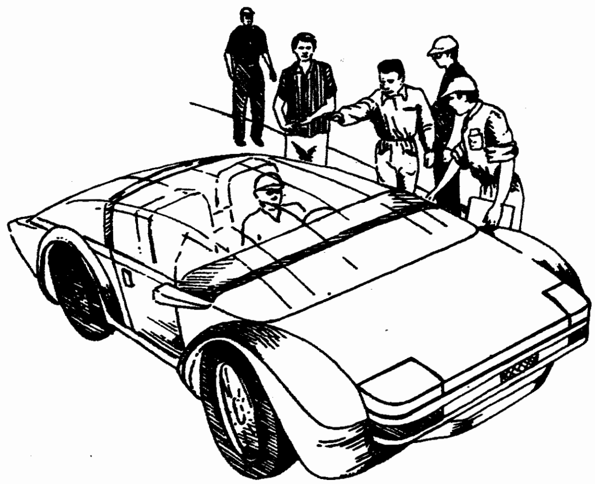
   d. Draw a graph for the average speed on the same coordinate system.



8. Use the distance-time graph at the right to answer each question.
   a. How fast was this car going during the first 0.5 hours? ____________
   b. How fast was this car going during the second 0.5 hours? ____________
   c. What was the average speed of the car during the 1-hour period?

   ____



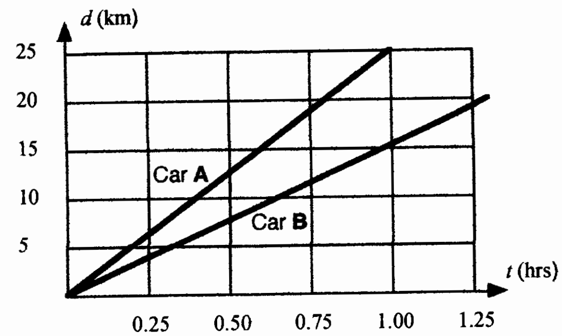


9. *a*. Which car is going faster?

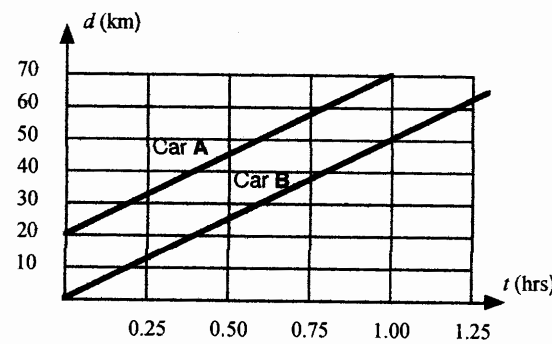
____

b. How much faster?

____



10. Study the graph at right and answer the questions that follow.



a. How fast is car *A* going?

____

b. How fast is car *B* going?

____

c. What do parallel lines indicate about speed on a distance-time graph?

____

d. What is the *d*-intercept of the graph for car *A*? ______________

What does this intercept indicate about the two cars?

____

11. Study the graph below and answer the questions that follow.

a. Which car is going faster?

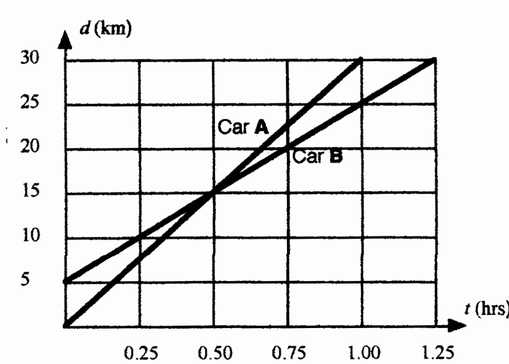
____

b. At what point do the two graphs intersect?

____

What is the physical significance of this point?

____



Week 4

Problem Solving

Problem Solving Strategies

1. Act it out
2. Draw a diagram
3. Make a table
4. Make a graph
5. Work backwards
6. Systematize the counting process
7. Look for a pattern
8. Find a rule
9. Make the problem simpler

Time

1. How many days is a million seconds? one million = 1,000,000
2. How many days is a billion seconds? one billion = 1,000,000,000
3. How many days is a trillion seconds? one trillion = 1,000,000,000

Checkerboard

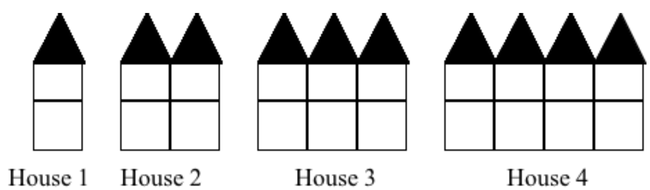
How many squares are there on an 8x8 checkerboard? Squares must follow the gridlines of the checkerboard. They can be any size and can overlap.

Develop a formula or rule for finding the number of squares on checkerboards of any dimension, (such as a 37x37 checkerboard)

Tower of Hanoi

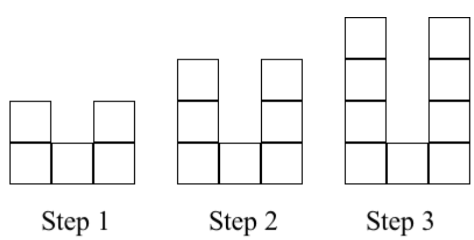
Do you know the game called the Tower of Hanoi? Model the game. What is the least number of moves for a 3-tower game? A 4-tower game? Can you generalize?

Houses



How many triangles and how many squares are needed for house 5?
How many triangles and how many squares are needed for house *n*?

"U"



Draw step 4.

Predict what the 10^{th} step of the pattern would look like. What is a rule or formula that can help us find the n^{th} step of the pattern?

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