Southern Africa
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MATHS
SENIOR LEVEL
UNISWA
SWAZILAND
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Grades 10-12 Priority Topics: Solids in Trigonometry and Matrices

Solids in Trigonometry

1. Pyramid $TABCD$ has a 20-cm square base $ABCD$. The edges that meet at T are 27 cm long. Make a diagram of $TABCD$, showing F, the point on $ABCD$ closest to T. To the nearest 0,1 cm, find the height TF.

2. (Continuation) Let P be a point on edge AB, and consider the possible sizes of angle TPF. What position for P makes this angle as small as it can be? How do you know?

3. (Continuation) Calculate the size of angle TAF.

4. The Great Pyramid of Giza is a square pyramid with a height of 138.8 meters and a base length of 230.4 meters. A bug walks in a straight line from the vertex of the pyramid to the midpoint of a side of the base and continues onto the horizontal surface of the ground. What angle did the bug make as she moved from the pyramid to the ground?

5. The base of a pyramid is the regular polygon $ABCDEFGH$, which has 14-cm sides. All eight of the pyramid's lateral edges, $VA, VB, ..., VH$, are 25-cm long. To the nearest centimeter, calculate the height of pyramid $VABCDEFGH$.

6. To the nearest tenth of a degree, calculate the size of the *dihedral angle* formed by the octagonal base and the triangular face VAB.

7. Dana takes a sheet of paper, cuts a 120-degree circular sector from it, then rolls it up and tapes the straight edges together to form a cone. Given that the sector radius is 12 cm, find the height and volume of this paper cone.

8. The base radius of a cone is 6cm, and the cone is 8cm tall. To the nearest square centimeter, what is the *lateral area* of the cone?

9. Infinitely many sectors can be cut from a circular piece of paper with a 12-cm radius, and any such sector can be fashioned into a paper cone with a 12-cm *slant height*.

   (a) Show that the volume of the cone produced by the 180-degree sector is larger than the volume of the cone produced by the 120-degree sector.

   (b) Find a sector of the same circle that will produce a cone whose volume is even larger.

   (c) Express the volume of the cone formed from this circle as a function of the central angle of the sector used to form it, then find the sector that produces the cone of greatest volume.

Matrices

1. A *matrix* can be used to display and process certain kinds of data. For example, Amin's batik factory has three different sizes of batiks: small, medium, and large. On Friday, the factory sold 10 smalls, 4 mediums, and 5 larges. On Saturday, they sold 3 smalls, 15 mediums, and 20 larges. These data are displayed in **A**, the 2×3 matrix below. Add labels above the columns and above the rows to help a reader remember what all the numbers mean.

$$\begin{bmatrix} 10 & 4 & 5 \\ 3 & 15 & 20 \end{bmatrix}$$

2. Create **B**, a 2×3 matrix, to display that Bandile's batik factory sold 4 smalls, 3 mediums, and 9 large batiks on Friday and 7 small, 3 medium and 8 large on Saturday.

3. (Continuation) Construct **T**, a 2×3 matrix, to display the total number of each type of shirt sold each day by both Amin and Bandile.

4. (Continuation) The matrix you created in the previous problem can also be formed through the process of *matrix addition* to create **A**+**B**. How do you think each number in the addition matrix could be formed from those in the first two matrices?

5. Construct a 2×3 matrix **M** that provides the mean number of each type sold by Amin and Bandile on the two days.

6. (Continuation) What did you do to each element of **T** in order to create matrix **M**? This is an example of the *product of a matrix and a scalar.*

7. Amin sells the small batiks for E30, the medium ones for E50, and the large ones for E100. Represent these data in a 3×1 matrix **P**. Label each row and column.

8. (Continuation) Create a matrix **R** whose rows represent days and whose columns represent the revenue brought in by all three types of batiks. What is the *order* of your matrix?

9. (Continuation) Matrix **R** can be thought of as the *product* of matrices **A** and **P**. How would you create the elements of a product matrix? What must be true about the orders of the matrices?

10. (Continuation) Amin and Bandile are considering merging their factories. If Bandile sells batiks for the same price as Amin, how much combined revenue would they have brought in? Represent your calculation as a product of two matrices.

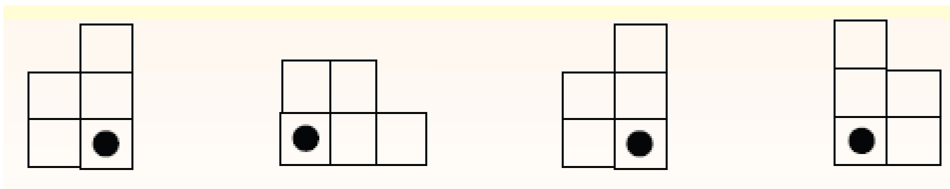
Transformations and Matrices

11. A *transformation* is a way to manipulate a point, line, or figure. The original object is called the *preimage* and the resulting one is known as the *image*. Using your own words, describe different types of transformations that maintain the size and shape of the original object.

12. Using your own words, describe a transformations which *do not* maintain the size of the original object.

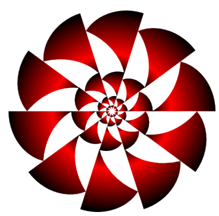
13. Using your own words, describe a transformations which *do not* maintain the shape of the original object.

14. Explain how the first figure below could be *transformed* into the following images.



15. (Continuation) A *rotation* is a transformation that moves points so that they stay the same distance from a fixed point, known as the *centre of rotation*. Which of the figures in question 14 is a rotation?

16. A figure has *rotational symmetry* if an outline of the turning figure matches its original shape. In other words, if the figure looks the same after a rotation, then the figure has rotational symmetry. Explain why the following figure has rotational symmetry.



17. (Continuation) How many times can the figure above be rotated so that it looks the same before it returns to the original orientation? This is the *order of symmetry*.

18. Is a rotation an *isometric transformation*, meaning one that maintains the original size and shape of the object, or a *nonisometric transformation*?

19. Consider the transformation which takes points (x, y) and moves them to $(-x, y)$. Take the unit square, which has vertices at $(0, 0)$, $(1, 0)$, $(1, 1)$ and $(0, 1)$, and apply this transformation. How would you describe this transformation in your own words?

20. Describe the effect of each of the following geometric transformations. To generate and test your hypotheses, transform some simple points.

    (a) $(x, y) \to (-y, -x)$

    (b) $(x, y) \to (0.6x - 0.8y, 0.8x + 0.6y)$

    (c) $(x, y) \to (-y, x)$

    (d) $(x, y) \to (-3x, -3y)$

21. (Continuation) Each transformation takes the general form $(x, y) \to (ax+by, cx+dy)$. For example, in problem 19, $a = -1$, $b = 0$, $c = 0$, and $d = 1$. What are the values of a, b, c, and d in the above problems?

22. (Continuation) We can use matrices to represent transformations. Explain the connection between $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ and the transformation $(x, y) \to (ax+by, cx+dy)$. Write the *coefficient matrix* for each of the transformation above.

23. The matrix $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$ represents the transformation which takes (x, y) to $(3x-4y, 4x+3y)$.

    (a) Apply the transformation to the unit square whose vertices are $(0, 0), (1, 0), (1, 1)$ and $(0, 1)$. In particular, notice what the images of the points (1,0) and (0,1) are and compare them with the entries in the columns of the coefficient matrix.

    (b) Confirm that the same results can be obtained by some matrix arithmetic: Calculate the products $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and interpret the results.

24. Calculate the products $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and interpret the results.

25. Find entries of the following matrices:

    (a) the 2×2 matrix $\mathbf{M}$ for the reflection across the line $y = x$.

    (b) the 2×2 matrix $\mathbf{R}$ for the 90-degree counterclockwise rotation about the origin.

    (c) the product $\mathbf{MR}$; what transformation does this represent?

    (d) the product $\mathbf{RM}$; what transformation does this represent?

    (e) the product $\mathbf{MM}$; what transformation does this represent?

26. Apply the transformation given by $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ to the unit square, which has vertices at $(0, 0), (1, 0), (1, 1)$ and $(0, 1)$. What shape does the image have?

27. (Continuation) Find the area of the image in terms of a, b, c and d.

28. Apply the transformation given by $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ to the unit square. What is its image? Notice that $ad - bc = 0$ for this matrix.

29. Take the unit square, which has vertices at $(0, 0), (1, 0), (1, 1)$ and $(0, 1)$, and apply the transformation which takes points (x, y) and moves them to $(x + 2y, y)$. How would you describe this *shear* in your own words?

30. (Continuation) Which points in the unit square remained the same after the transformation? These points are known as *invariant*.

31. (Continuation) Find the area of the image that resulted from the shear transformation.
32. (Continuation) Is a shear an isometric transformation or a non-isometric one? Does it maintain the size of the preimage? What about its shape?
33. How far did point (1,1) move under the shear in question 29? How far was (1,1) from the *invariant line*, the line of points which did not move? The ratio of the distance a point moved to its original distance from the invariant line, provided that it was not on the invariant line to begin with, is known as the *shear factor*.
34. Transform the rectangle with vertices $(0,0),(0,1),(1,2)$ and $(2,0)$ by using a shear with a shear factor of 2 and the y-axis as an invariant line.
35. Let $A=(-1,0)$, $B=(1,0)$ and $C=(0,1)$. Apply a shear with a shear factor of 1 and $y=-2$ as the invariant line.
36. Take the unit square, which has vertices at $(0,0),(1,0),(1,1)$ and $(0,1)$ and apply the transformation (x,y) goes to $(x,3y)$. Describe this transformation in your own words.
37. (Continuation) What is the invariant line in the above transformation?
38. (Continuation) Find the area of the image that resulted from the *stretch* transformation in question 36.
39. (Continuation) Is a stretch an isometric transformation or a non-isometric one? Does it maintain the size of the preimage? What about its shape?
40. Let $P=(1,1)$. What is P', the image of P under stretch in question 36? How far is P' from the invariant line? How far is P? The ratio of the distance an image point is from the invariant line to the distance the primage is from the invariant line, provided that the preimage was not on the invariant line to begin with, is known as the *stretch factor*.
41. Transform the rectangle with vertices $(0,0),(0,1),(1,2)$ and $(2,0)$ by using a stretch with a stretch factor of 2 and the y-axis as an invariant line.
42. Let $A=(-1,0)$, $B=(1,0)$ and $C=(0,1)$. Apply a stretch with a stretch factor of 1 and $y=-2$ as the invariant line.

Prelude: Workshop focus considerations brad.uy@gmail.com

10-11 Statistics (the histogram, scatter plots), Probability, Congruence and Similarity

Statistics--reasoning why a particular average should be used; the histogram

I. Data
  A) Classifying
    1. Univariate v bivariate data
      a) for univariate: we can use 5 number summary, box and whisker diagrams
        1. And ogives to represent distribution
      b) for bivariate data, we can use scatter plots and linear regression lines

II. Univariate data analysis, perhaps even a histogram
  A) Measures of central tendency: mean, median, and mode
    1. meaning of the mean
    2. purpose of the median
      a) robust, relatively unaffected by outliers
    3. mode, catering to groups--common questions
  B) Histograms allow us to visually examine the distribution of the data
    1. distributions commonly are….
      a) uniform
      b) skewed the the left or right; in reference to the tail; shift of mean
      c) normal
        i. Empirical rule
          a) 1, 2, and 3 standard deviations about the mean encompass .68, .96, and 1 data
    2. they allow us to visualize distribution

III. Bivariate data using scatter plots
  A) Purpose
    1. Visually explore trends, patterns, correlations
  B) Given a scatterplot, we can calculate the correlation coefficient (as a measure of validity of below)
  C) linear regression formula, for the use of …

S1 Investigation. Given the data values 8, 10, and 12. (Try to plot on a number line….) If we add five to every data point, is mean affected? How is it affected? (Before you do any calculations, form a hypothesis about the following questions…) Are variance and standard deviation affected? If so, how are they affected? What does this tell you about what variance and standard deviation are measuring?

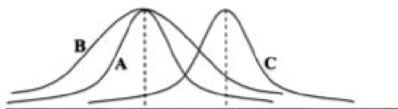
Again, given the data 10, 15, 20. (Again, try to plot on a number line, well...kind of attempt it….) Now we scale down every data point by a factor of 5. Is mean affected? How is it affected? (Before you do any calculations, form a hypothesis about the following questions...) Are variance and standard deviation affected? If so, how are they affected? What does this give you a sense of what variance and standard deviation are measuring?

QUESTION 4

The Grade 10 classes of three schools wrote a term test. All three schools have the same number of learners in Grade 10. The results of the tests have been summarised in the table below.

	SCHOOL A	SCHOOL B	SCHOOL C
Mean	9,8	9,8	14,8
Standard deviation	2,3	3,1	2,3

The distribution of the results is shown in the diagram below.



4.1 In which school (A, B or C) is the majority of the results more widely spread around the mean? Give a reason for your answer. (2)

4.2 What is the difference in the spread around the respective means of the marks in School A and School C? (1)

4.3 Explain how the marks of School A must be adjusted to match the marks of School C. (2)

4.4 If each mark in School C is lowered by 10%, explain the effect it will have on the mean and standard deviation of this school. (2)
[7]

S2

1. Revise symmetric and skewed data.
2. Use statistical summaries, scatterplots, regression (in particular the least squares regression line) and correlation to analyse and make meaningful comments on the context associated with given bivariate data, including interpolation, extrapolation and discussions on skewness.

<u>Example:</u>

The following table summarises the number of revolutions x (per minute) and the corresponding power output y (horse power) of a Diesel engine:

x	400	500	600	700	750
y	580	1030	1420	1880	2100

1. Find the least squares regression line $y = a + bx$ (K)
2. Use this line to estimate the power output when the engine runs at 800m. (R)
3. Roughly how fast is the engine running when it has an output of 1200 horse power? (R)

calculate corresponding linear correlation coefficient as well. Any efficient methods to share…

Probability--dependent and independent events

I. Introduction
   A) constructing a sample space
      1. using tables and tree diagrams to help
   B) Definition of probability
      1. two coins; two dice and their sums; graduate to three births
      2. evaluating respective probabilities
   C) Probability is a number--sliding scale
      1. Placing words, impossible, even chance, certain, somewhat likely, somewhat unlikely, “possible”

II. Analytic aids
   A) Venn diagrams
   B) Tree diagrams
      1. following through branches with multiplication
         a) adding outcomes in the end
            i. application of addition rule

III. Everyday terms, meaning of independent and dependent
   A) give examples of independent and dependent events
   B) If the occurrence of one event affects the probability of the other event...
   C) Note: if we replace, we consider the selections a case of ____________ events
      If we do not replace, we consider the selections a case of _____________ events.

IV. Multiplication rule is what we apply in cases of independent and dependent events.
   A) Relevant as we're talking about a sequence of separate events in trials
      1. easy if they are independent
      2. adjust accordingly if they are dependent
   B) underscores that "and" in context of probability implies what operation

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P0 before we discuss probability, let's discuss how we conceive of probability…. Sliding scale probability intuitive sense of probability as a sliding scale--ask them probability that they'll eat something green tonight;encounter a bicycle on the way home; get sent to the principal later today; get pooped on by a bird; etc. They can even imagine their own events and trade questions with each other.
--we also need to define or remind our learners of the definition of how to calculate probability.

P1 What is the probability that the first child born will be a girl; and the second will also be a girl? Are these independent or dependent events.
--how can we represent this using a visual aid to justify the multiplication rule?

P2 What is the probability of guessing on a true and false question first, and a multiple choice question next (with choices a] through e]); and getting the first answer *and* second answer both correct?
--Are these independent or dependent events?

P3 Let's say we have only 4 geometric figures made of plastic to choose from. A sphere, a pyramid, a cube, and a cylinder. Without replacement, what is the probability of me randomly selecting a cube followed by a pyramid? Without replacement, what is the probability of me randomly selecting a pyramid, followed by another pyramid?

--Now with replacement, what is the probability of me randomly selecting a pyramid, followed by another pyramid? (Is this last selection an example of independent or dependent events?)

P4 For the following questions, determine first whether events A and B are independent or dependent;
then, find P(A and B), meaning the probability that events A and B both occur in that particular order.
(this exercise reinforces the different types of events, and how the probability is calculated)

A: When a baby is born, it is a girl.
B: When a single die is rolled, the outcome is a 6.
…
A: When you have two numbered balls to choose from, one with 20 and the other with 50; one selects 50 first and does not replace it.
B: Then one selects a 20.
…
A: When you have two numbered balls to choose from, one with 20 and the other with 50; one selects 50 first and then puts it back in the bag.

B: Then one selects a 20.
...
A: When a month is randomly selected from a calendar, then ripped out and destroyed, it is July.
B: When a different month is randomly selected from this same calendar, then ripped out and destroyed, it is November.
…
A: When the first digit (0-9) of a four-digit lottery number is guessed at by someone buying a ticket, it is the same first digit that is later drawn in the official lottery.
B: When the second digit of a four-digit lottery number is guessed at by someone buying a ticket, it is the same second digit that is later drawn in the official lottery.

P5 Let's say I know that if I pour boiling hot water into any mug, then the probability that a mug will break is 80%. What is the probability of pouring boiling water into two mugs, and having them both break?
What is the probability of pouring boiling water into two mugs, and having them both survive?

P6 A battery-powered alarm clock works properly 90% of the time when set by an average person. If I set 3 alarms, what is the probability that they will all work properly? What is the probability that they will all fail? What is the description of the compliment of all 3 alarms failing?
http://www.dg58.org/cms/lib02/IL01001925/Centricity/Domain/904/9%20-%207%20independent_vs_dependent.pdf

NAME ____________________________
INTERACTIVE ALG/GEOM II **Probability**

Be Sure to Show ALL Your Work!!!

Special Note: Reduce all fractions to lowest terms.

1. **A card is chosen at random from a deck of 52 cards. It is then *replaced* and a second card is chosen. What is the probability of choosing a Jack first and a 3 second?**

2. **What is the probability that from a normal 52 card deck, you randomly draw a 3, and then *without replacing* the 3, you draw the Queen of Hearts?**

3. **A jar contains 6 red balls, 3 green balls, 5 white balls and 7 yellow balls. Two balls are chosen from the jar, with replacement. What is the probability that both balls chosen are green?**

4. **A box contains a penny, a nickel, and a dime. Find the probability of choosing a dime first and then, without replacing the dime, choosing a penny.**

5. **A coin is tossed and a single 6-sided die is rolled. Find the probability of landing on the head side of the coin and rolling 3 on the die.**

6. **The teacher of a class that contains 12 boys and 16 girls needs to pick two volunteers. She randomly selects one student, and then selects another student from the class. Find the probability that**

   a. she chooses a girl first, then a boy **b. she chooses two boys**

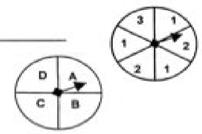
http://www.northlandprep.org/wp-content/uploads/2014/08/Compound-Probability-WS.pdf

Probability with Compound Events (Independent and Dependent)
Practice

Describe the events by writing **I** for *independent event* or **D** for *dependent event.*

1. Ann draws a colored toothpick from a jar. Without replacing it, she draws a second toothpick. _______
2. John rolls a six on a number cube and then flips a coin that comes up heads. _______
3. Susie draws a card from a deck of cards and replaces it. She then draws a second card. _______
4. Seth draws a colored tile from a bag, replaces it; draws a second tile from the bag, replaces it; and then draws a tile a third time from the bag. _________
5. You draw a red marble from a bag, and then another red marble (without replacing the first marble)? ____

Using the two spinners, find each **compound** probability.



6. P(A and 2) _________ 7. P(D and 1) _________ 8. P(B and 3) _________

9. P(A and not 2) _________

A box contains 3 red marbles, 6 blue marbles, and 1 white marble. The marbles are selected at random, one at a time, and are **not replaced**. Find each **compound** probability.

10. P(blue and red) _________ 11. P(blue and blue) _________ 12. P(red and white and blue) _______

13. P(red and red and red) __________ 14. P(white and red and white) __________

A	R	R	R	C
A	R	R	R	C
E	E	E	E	C

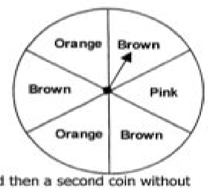
Suppose that two tiles are drawn from the collection shown at the right. The first tile is replaced before the second is drawn. Find each **compound** probability.

15. P(A and A) __________ 16. P(R and C) _________ 17. P(A and not R) __________

Suppose that two tiles are drawn from the same collection shown above. The first tile is **not** replaced before the second is drawn. Find each **compound** probability.

18. P(A and A) __________ 19. P(R and C) ___________ 20. P(A and not R) __________

Use the spinner to the right for the next two problems.



21. If you spin the spinner twice, what is the probability of spinning orange then brown? ___________

22. If you spin the spinner twice, what is the probability of spinning brown both times? ___________

23. Kevin had 6 nickels and 4 dimes in his pocket. If he took out one coin and then a second coin without replacing the first coin ---
    (a) what is the probability that both coins were nickels? ___________
    (b) what is the probability that both coins were dimes? ___________
    (b) what is the probability that the first coin was a nickel and the second a dime? ___________

ANSWERS

Probability with Compound Events (Independent and Dependent)
Practice

1. D 2. I 3. I 4. I 5. D

6. $\frac{1}{2} \times \frac{2}{6} = \frac{1}{12}$
7. $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$
8. $\frac{1}{4} \times \frac{1}{6} = \frac{1}{24}$
9. $\frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$
10. $\frac{6}{10} \times \frac{3}{9} = \frac{1}{5}$
11. $\frac{6}{10} \times \frac{5}{9} = \frac{1}{3}$
12. $\frac{3}{10} \times \frac{1}{9} \times \frac{6}{8} = \frac{1}{40}$
13. $\frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} = \frac{1}{120}$
14. $\frac{1}{10} \times \frac{3}{9} \times \frac{6}{8} = \frac{1}{40}$
15. $\frac{2}{15} \times \frac{2}{15} = \frac{4}{225}$
16. $\frac{6}{15} \times \frac{3}{15} = \frac{2}{25}$
17. $\frac{2}{15} \times \frac{9}{15} = \frac{2}{25}$
18. $\frac{2}{15} \times \frac{1}{14} = \frac{1}{105}$
19. $\frac{6}{15} \times \frac{3}{14} = \frac{3}{35}$
20. $\frac{2}{15} \times \frac{8}{14} = \frac{8}{105}$ ($\frac{8}{14}$ because A also now removed)
21. $\frac{2}{6} \times \frac{3}{6} = \frac{1}{6}$
22. $\frac{3}{6} \times \frac{3}{6} = \frac{1}{4}$
23. (a) $\frac{6}{10} \times \frac{5}{9} = \frac{1}{3}$
    (b) $\frac{4}{10} \times \frac{3}{9} = \frac{2}{15}$
    (c) $\frac{6}{10} \times \frac{4}{9} = \frac{4}{15}$

Congruence and Similarity

I. Definitions of congruence and similarity, considering parts of 2-D figures (angles and sides)

A) Figures are congruent when …

B) Figures are similar when…

1. topic of proportionality, illustrating it...
2. scale factor vs scale ratio

C) Informal definitions using terms shape and size

- Learners should recognize that two or more figures are congruent if they are equal in all respects i.e. angles and sides are equal.
- Learners should recognize that two or more figures are similar if they have the same shape, but differ in size i.e. angles are the same, but sides are proportionally longer or shorter. Similar figures are further explored when doing enlargements and reductions. Refer to "Clarification Notes" under 3.4 Transformation Geometry.

II. Lab / demo / illustration of congruence and similarity, helping us to understand and recognize difference in concepts

A) Flashlights and plastic or glass, or cutouts

B) Can images be rotated or flipped and still preserve similarity or congruence?

C1 True or false? If two rhombuses have sides which are proportional (say, with a scale ratio of 1:2), then the two rhombuses are similar.

Discuss...

Related query: true or false? If the sides of two figures are proportional, then they are similar.

T or F? If two figures are similar, then they are congruent.

If two figures are congruent, then they are similar.

- Comparing rhombii with sides proportional, learners can ascertain that having sides proportional does not necessarily imply that the corresponding angles will be equal. So only having sides of proportional length is not a sufficient condition for similarity

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III. Exploring effects of enlargements (or reductions) of similar figures

A) From Caps

Enlargements and reductions

- Draw enlargements and reductions of geometric figures on squared paper and compare them in terms of shape and size

Enlargements and reductions

- Use proportion to describe the effect of enlargement or reduction on area and perimeter of geometric figures
- Investigate the co-ordinates of the vertices of figures that have been enlarged or reduced by a given scale factor

B) Proportions, commonly written as ratios, which we may refer to as scale ratio

C) Effects on perimeter and area (and by extension, volume for 3-D shapes)

C2 Lab exploration: Let's start with a square. Given a square with side length of 10 units. Draw it. Calculate The perimeter.

	Scale factor	Perimeter	Original perimeter has been multiplied by ___? = (Perim of new figure) / (**Perim of original**) = Perimeter-change multiplier
image	1/2		
Original shape	**1**		
image	2		
image	3		
image	4		

image	5		

Let’s say we enlarge this square by a factor of 2. What is the length of each side? Calculate the perimeter and compare it to the original.

Let’s say we enlarge the original square by a factor of 3. What is the length of each side? Calculate the perimeter and compare it to the original.

What general, universal rule can we develop?

(If you enlarge a figure by a factor of ___, then the perimeter will change by a factor of ___….
If the scale factor is __, then the perimeter will change by a factor of __.)

Make educated guesses to fill in the rest of the table. Check your answers.

What is the perimeter of the imaged square if the scale factor is ½? Confirm your answer.

IV. Discussion of perimeters of similar figures

A) This exercise may be repeated for a rectangle of dimensions 2 units by 4 units.

B) Or an obtuse triangle with side lengths, 4, 6, and 8.

C) Note we haven’t discussed proportions yet. But we can say that the proportion (ratio) of perimeters of similar shapes is equal to the proportion (ratio) of their corresponding sides (this is the scale ratio).

1. For example, let’s say we have similar octagons. The scale ratio is 1:4. If the perimeter smaller octagon is 20 cm, what is the perimeter of the larger octagon.

V. Moving on to effect of enlargement on area

A) Question to motivate discussion: if we enlarge a figure by a factor of 2, will the area increase by a factor of 2?

B) Any guesses? Counterexamples?

C3 Lab: Let’s start with a square. Given a square with side length of 4 units. Draw it. Calculate the area.

	Scale factor	Area	Original area has been multiplied by ___? = (Area of new figure) / (**Area of original**) = Area-change multiplier
image	1/2		
Original shape	**1**		**1**
image	2		
image	3		
image	4		
image	5		

Let’s say we enlarge this square by a factor of 2. What is the length of each side? Calculate the area and compare it to the original.

Let’s say we enlarge the original square by a factor of 3. What is the length of each side? Calculate the area and compare it to the original.

What general, universal rule can we develop?

(If you enlarge a figure by a factor of ___, then the area will change by a factor of ___….
If the scale factor is __, then the area will change by a factor of __.)

Make educated guesses to fill in the rest of the table. Check your answers.

What is the area of the imaged square if the scale factor is ½? Confirm your answer.

--This exercise may be repeated for a rectangle of dimensions 2 units by 4 units.

--Or a right triangle with side lengths 6, 8, 10.

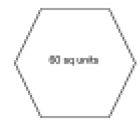
C4 Note we haven't discussed proportions yet. But we can say that the proportion (ratio) of areas of similar shapes is equal to the square of the proportion (ratio) of their corresponding sides (this is the scale ratio).

--For example, say we have two squares with a scale ratio of 1:4. The smaller square has an area of 8 sq. units. What is the area of the larger square?

--we can prove this by finding the side lengths of the smaller and larger squares....

C5 A hexagon has an area of 60 sq units.

What will the area of the new hexagon be if it is enlarged by a factor of 3?



VI. Prefiguring rules for similar figures and their volume

- A) Explore...If we enlarge a cube by a factor of 2, will the volume increase by a factor of 2?
  1. Test with numbers. Say start with a cube of side length 5 cm. Solve...
  2. Generate a rule

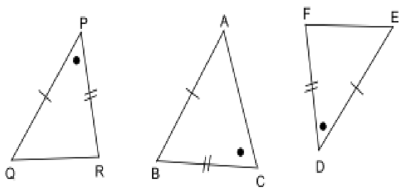
VII. Dilations and coordinates of images??

II. Common Postulates and definitions

- A) Algebraic properties
  1. Reflexive property of equality (or of congruence)
- B) Definitions
  1. Definition of perpendicular lines
  2. CPCTC (corresponding parts of congruent triangles are congruent)
     - a) special case of definition of congruence
  3. Definition of similarity
- C) Postulates
  1. Segment addition postulate
  2. Angle addition postulate
  3. The sum of the angles of a triangle is 180^0
- D) Theorems
  1. For proving triangles congruent
     - a) SSS, ASA, SAS, AAS, and HL
       1. Note that AAA and ASS are not allowed….why
  2. Given parallel lines cut by a transversal
     - a) corresponding angles are congruent (postulate)
     - **b) alternate interior angles are congruent (try prove this)**
     - c) same side interior (co-interior) angles are supplementary
     - d) given these conditions, these are also ways to prove lines are parallel
  3. Vertical angles are congruent
  4. In a triangle, angles opposite congruent sides are congruent (isosceles triangle theorem)
  5. All radii of a circle are congruent (possibly by definition)

6. AAA similarity theorem for triangles

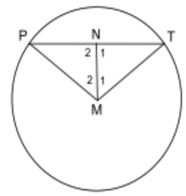
10.1



Which triangle is congruent to ΔPQR?

Statement	Reason

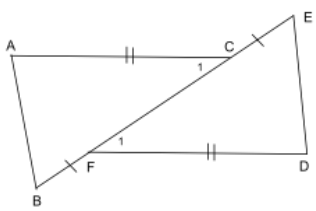
10.2 In the given figure, P and T are points on a circle with centre M. N is a point on a chord PT such that $MN \perp PT$.



Prove that $PN = NT$.

Statement	Reason

10.3



In the above diagram, $AC = DF$, $AB = DE$ and $BF = CE$.

10.3.1 Prove that $BC = EF$.

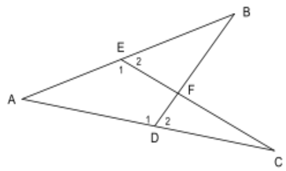
Statement	Reason

(2)

10.3.2 Prove that $\Delta ABC \equiv \Delta DEF$.

10.3.3 Why is angle B congruent to angle E? 10.3.4 What is the relationship between line AB and line ED?

10.4



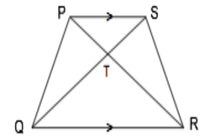
In the figure, $\hat{B} = \hat{C}$, $AD = 9$ cm, $AE = 7$ cm and $CE = 21$ cm.

10.4.1 Prove that $\Delta ABD \,|||\, \Delta ACE$.

Statement	Reason

(6)

1.8 In the figure below, $PS \parallel QR$. Which ONE of the following statements is true for this figure?



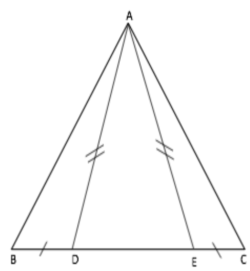
A ΔPTS ≡ ΔPQT

B ΔPTS ≡ ΔRTQ

C ΔPTS ||| ΔSRT

D ΔPTS ||| ΔRTQ

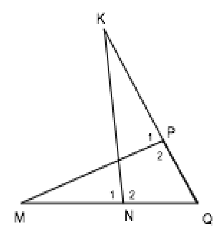
8.2 In ΔABC, D and E are points on BC such that $BD = EC$ and $AD = AE$.



8.2.1 Why is $BE = CD$?
____ (1)

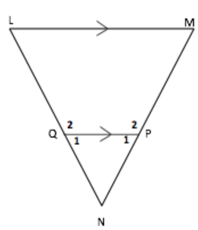
8.2.2 Which triangle is congruent to ΔABE ?

8.3 In the figure below ΔKNQ and ΔMPQ have a common vertex Q.
P is a point on KQ and N is a point on MQ.
$KQ = MQ$ and $PQ = QN$.



Prove with reasons that $\Delta KNQ \equiv \Delta MPQ$.

8.4 In ΔNML below, P and Q are points on the sides MN and LN respectively such that $QP \parallel LM$.
$MN = 16$ cm, $QP = 3$ cm and $LM = 8$ cm.



8.4.1 Complete the following (give reasons for the statements):
Prove with reasons that $\Delta QPN \,|||\, \Delta LMN$.

In ΔQPN and ΔLMN

1. $\hat{N}$ = …………… ………………………………

2. $\hat{P}_1$ = …………… ………………………………

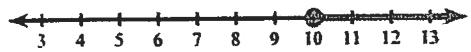
3. $\hat{Q}_1$ = …………… ………………………………

∴ ΔQPN ||| Δ……… ……………………………… (4)

8.4.2 Hence, calculate the length of PN.

Solve each inequality and graph its solution.

1) $2x + 4 \geq 24$



$x \geq 10$

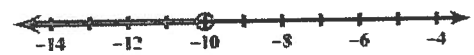
2) $\frac{m}{3} - 3 \leq -6$

$m \leq -9$

3) $-3(p + 1) \leq -18$

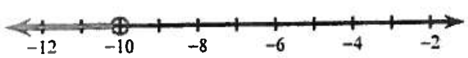
$p \geq 5$

4) $-4(-4 + x) > 56$



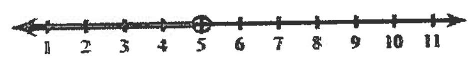
$x < -10$

5) $-b - 2 > 8$



$b < -10$

6) $-4(3 + n) > -32$

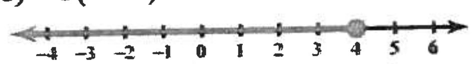


$n < 5$

7) $4 + \frac{n}{3} < 6$

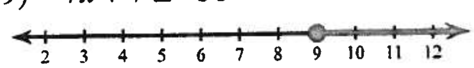
$n < 6$

8) $-3(r - 4) \geq 0$



$r \leq 4$

9) $-7x + 7 \leq -56$

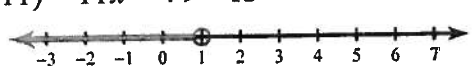


$x \geq 9$

10) $-3(p - 7) \geq 21$

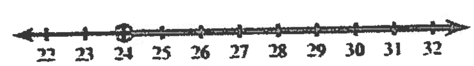
$p \leq 0$

11) $-11x - 4 > -15$



$x < 1$

12) $\frac{-9 + a}{15} > 1$



$a > 24$

Graphing Linear Inequalities in the Plane

Suppose we wish to graph $x + y \leq 10$. The standard procedure is to first graph the boundary line as sown in figure 1.

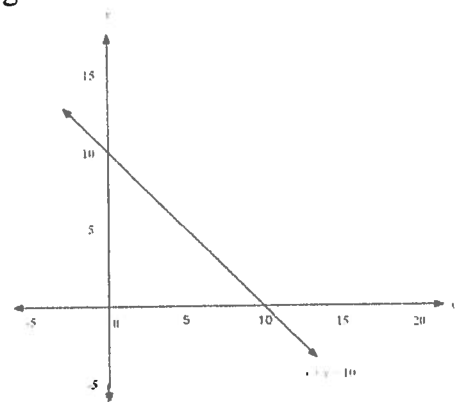


Fig. 1 Boundary line

The next step is to find a test point and see if it makes the inequality true or false. I have found that even though it seems unnecessary, students are more successful and more comfortable when they select two points, one on one side of the line and one on the other side. They must find that one point has coordinates that make the inequality true and the other point has coordinates that make the inequality false. They can then shade the area that has the point with coordinates that make the inequality true.

For example (3, 2) means $3 + 2 \leq 10$ is true while (10, 10) means $10 + 10 \leq 10$ is false, so the region below and including the line, shown in figure 2, is the solution.

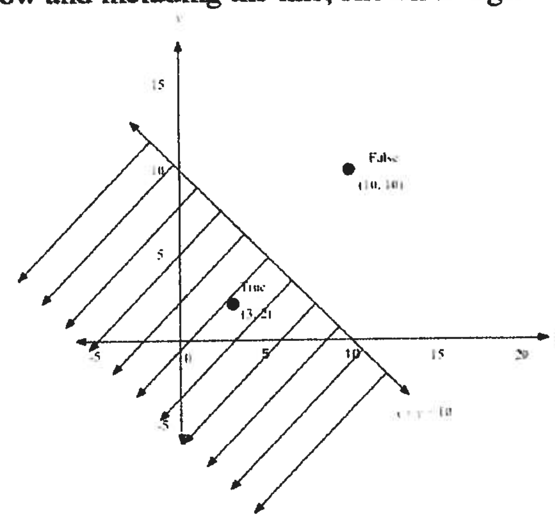
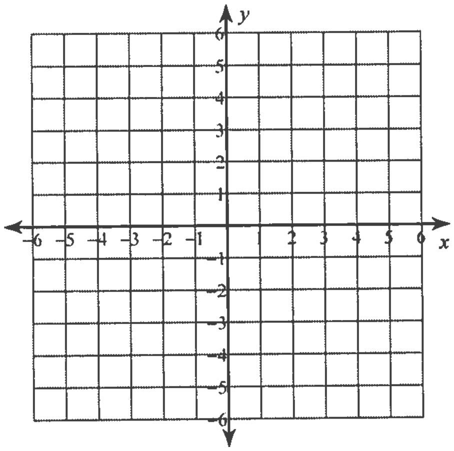


Fig. 2

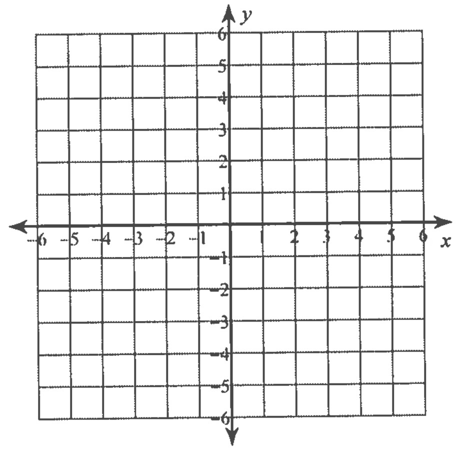
Of course one may actually shade the region. When the student realizes that only one point need to be tested, then it is no longer necessary to test two points, but this should be done only when the student is ready.

WEEK 3

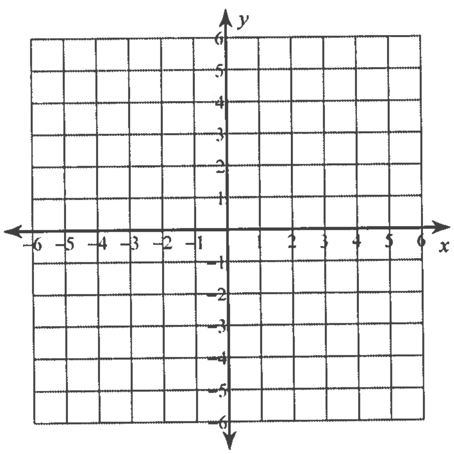
7) $x < -5$



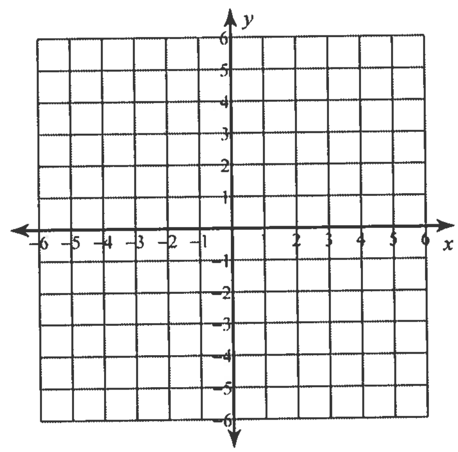
8) $y \le \frac{4}{3}x - 4$



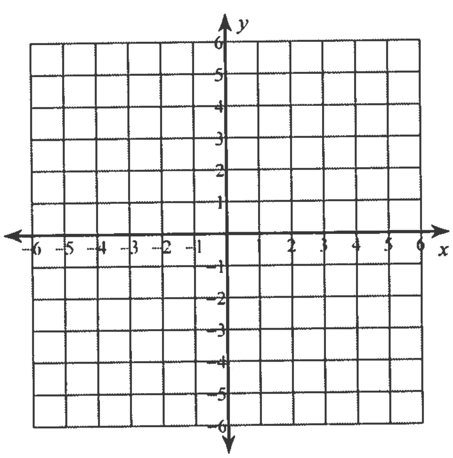
9) $3x - 2y < 10$



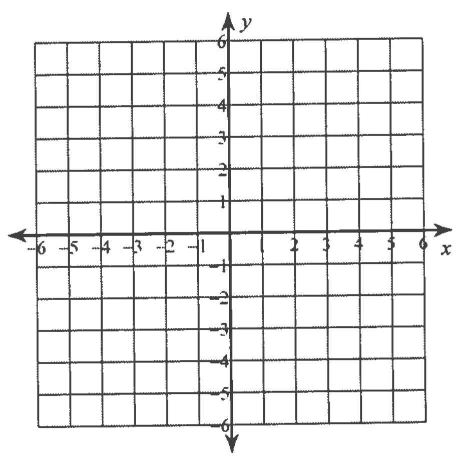
10) $5x - 3y \le -15$



11) $y \ge 4$

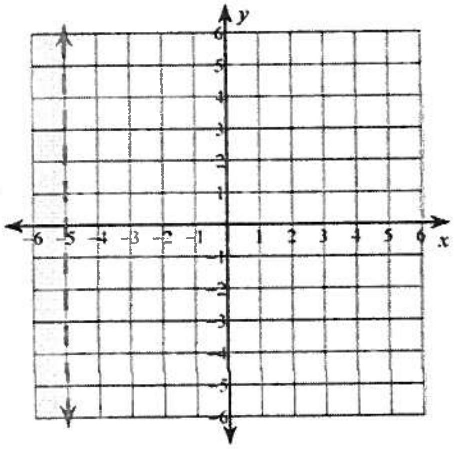


12) $x - y > 2$

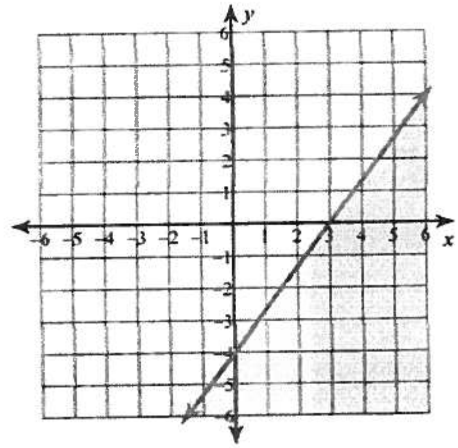


WEEK 3

7) $x < -5$



8) $y \le \frac{4}{3}x - 4$



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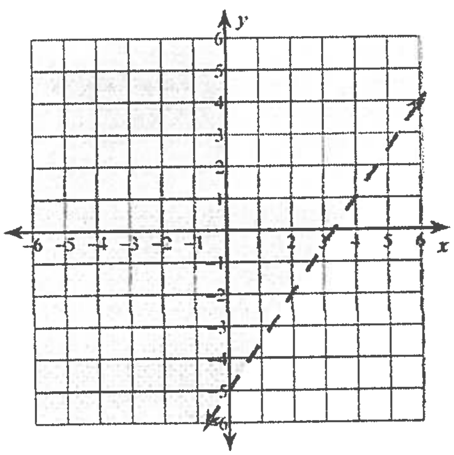
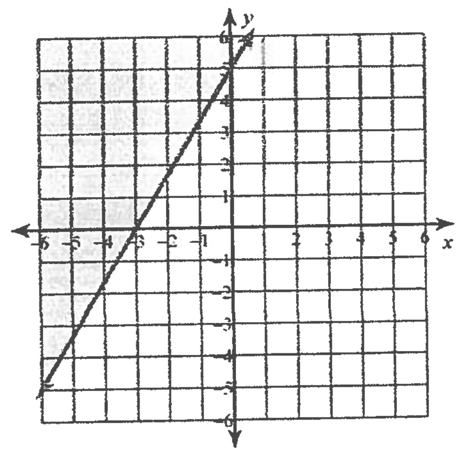
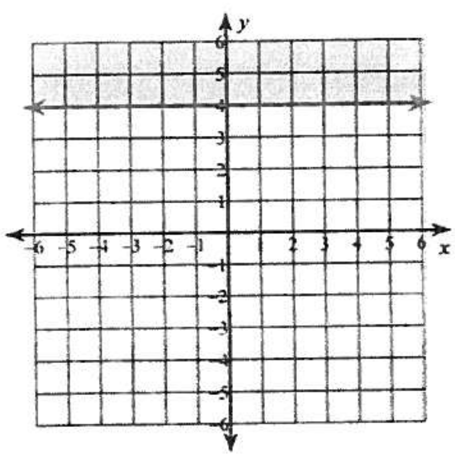


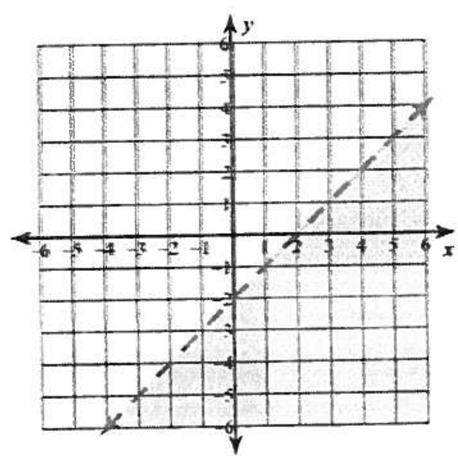
10) $5x - 3y \le -15$



11) $y \ge 4$



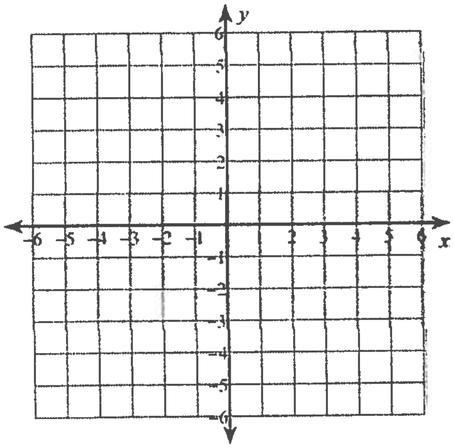
12) $x - y > 2$



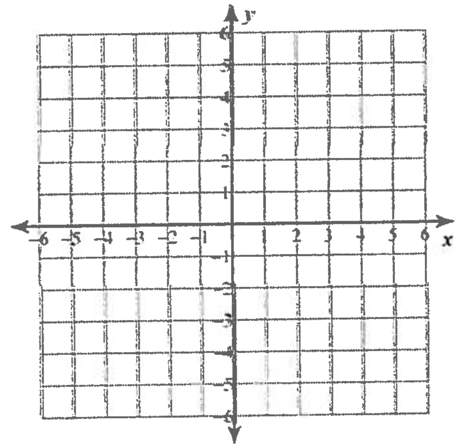
Sketch the graph of each linear inequality.

WEEK 3

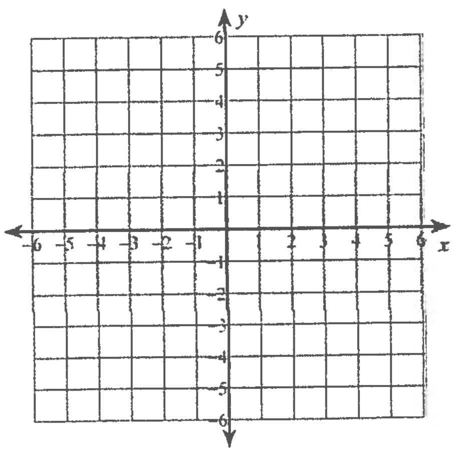
1) $y \geq -3x + 4$



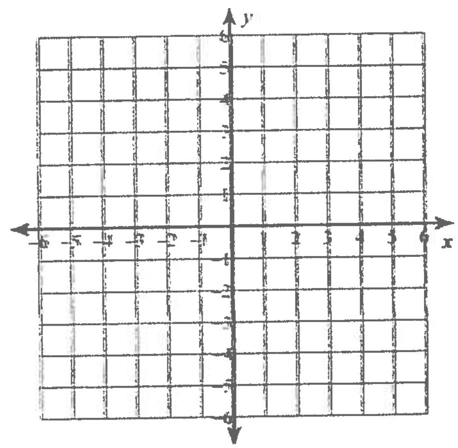
2) $y \leq \frac{3}{5}x - 5$



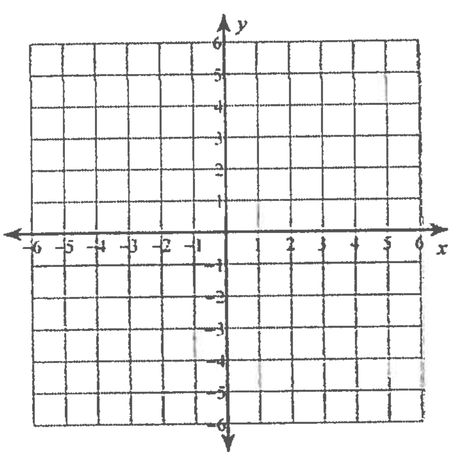
3) $y > -x - 5$



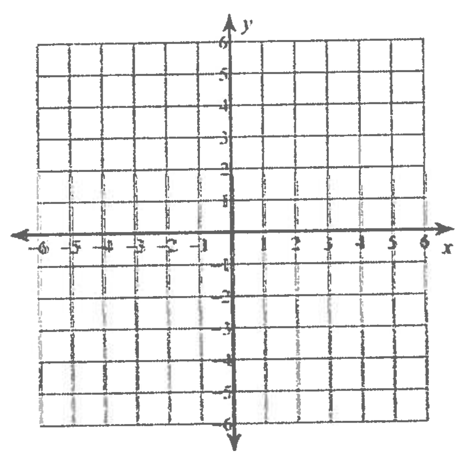
4) $y > -4$



5) $y > 2x - 5$

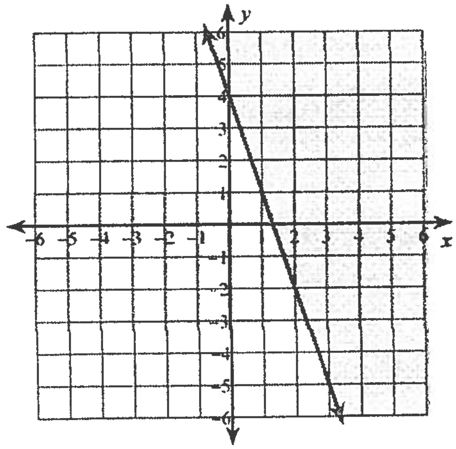


6) $y \geq \frac{7}{4}x + 2$

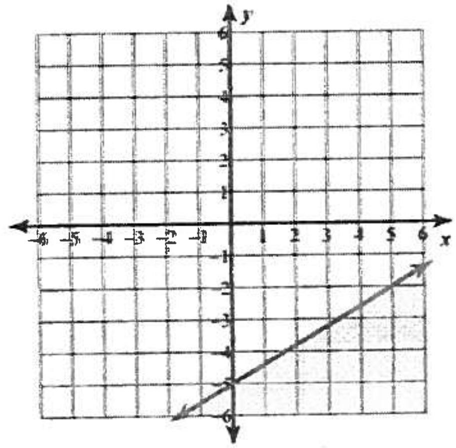


Sketch the graph of each linear inequality.

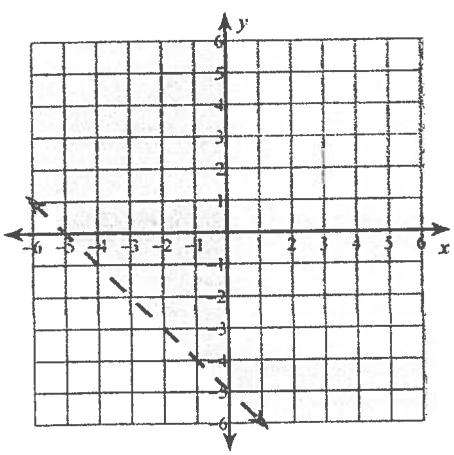
1) $y \geq -3x + 4$



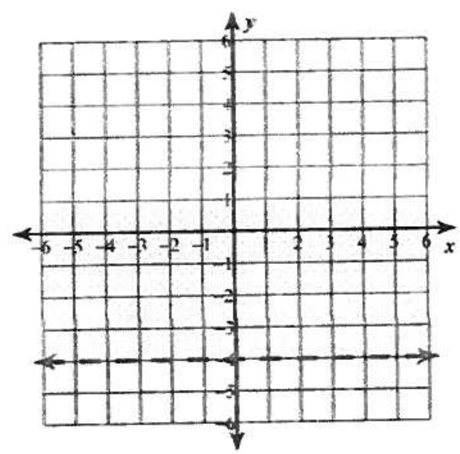
2) $y \leq \frac{3}{5}x - 5$



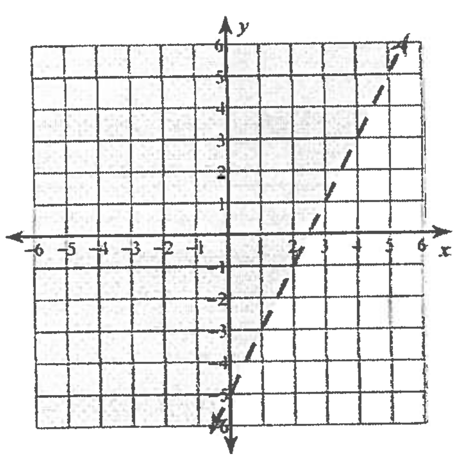
3) $y > -x - 5$



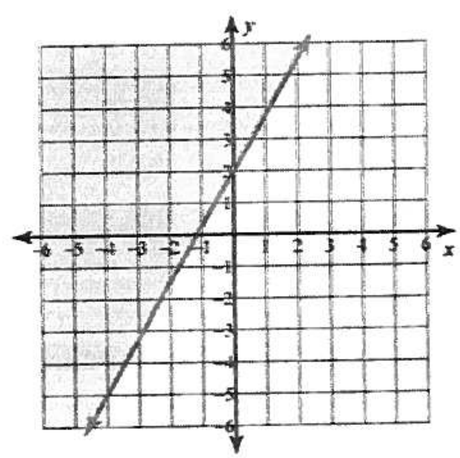
4) $y > -4$



5) $y > 2x - 5$



6) $y \geq \frac{7}{4}x + 2$



1. What do the below symbols mean, when possible (8 pts)

$\in$

$\cup$

$\cap$

$\neq$

n(P)

$\notin$

$\subseteq$

$\varnothing$

2. $A = \{2,3,4,5\}$, $B = \{1,3,4,6,7,8\}$ (4 pts)

a. Find $A \cup B$

b. find n($A \cup B$)

c. Find $A \cap B$

d. find n($A \cap B$)

3. Define the below symbols and give examples of what they are (4 pts)

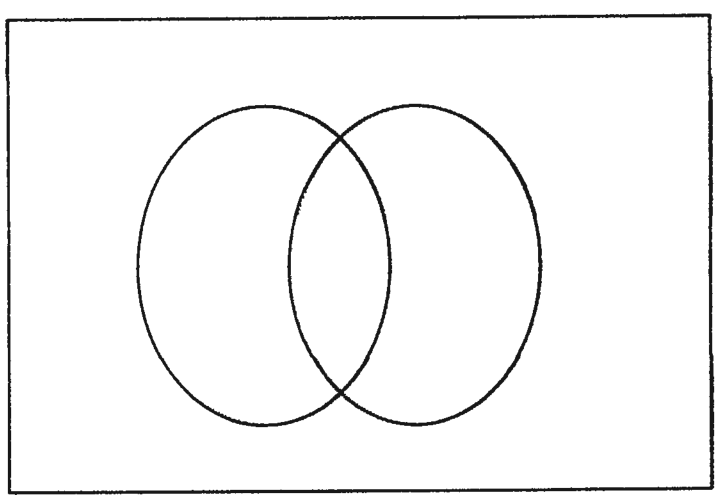
a. N:

b. Q:

c. R:

d. Z:

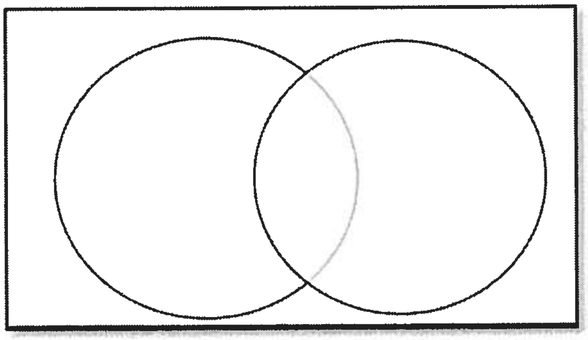
4. In the classroom there are 20 students, 9 students have brown hair, 11 students have brown eyes, 4 students have neither. Fill in the venn diagram to represent the problem. Show work and how you got each value. (4 pts)



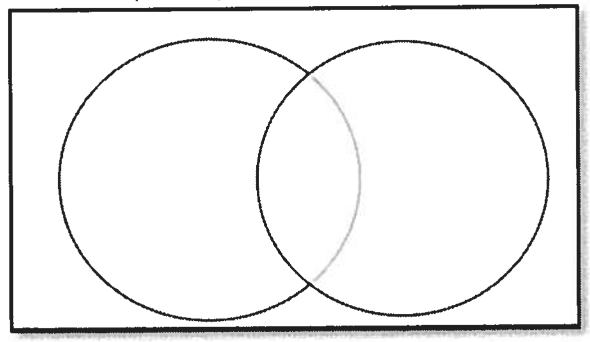
WEEK 3

5. Shade the region that is represented in the given set. (8 pts)

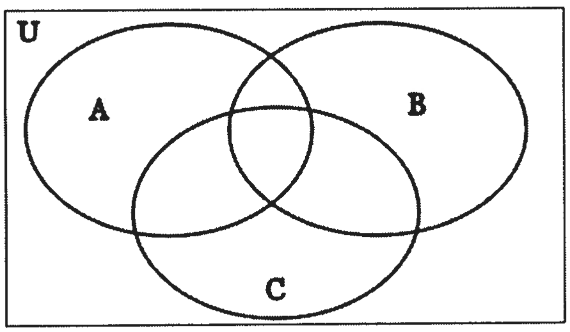
a. $P \cup Q'$



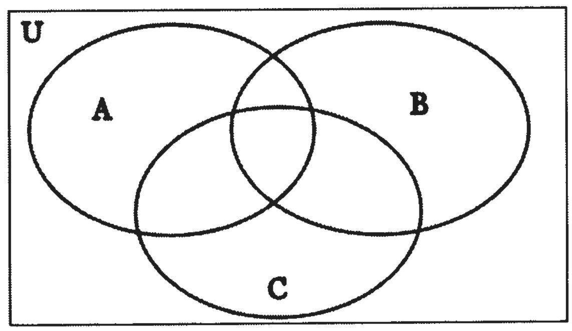
b. $(P \cap Q)'$



c. $(A \cup B) \cap C$



d. $A \cap (B' \cap C)$



6. $U = \{4,5,6,7,8,9,10],\ F = \{4,5,6,7\}\ and\ G = \{6,7,8,9\}$ (2 pts)

a. Draw a Venn Diagram for F, G, and U.

Bonus: +3

Suppose U={0,1,2,3,4,5 …. ,20}, F={$factors\ of$ 24}, M={$multiples\ of$ 4}

a. List the sets

i. F

ii. M

iii. M'

iv. $F \cap M$

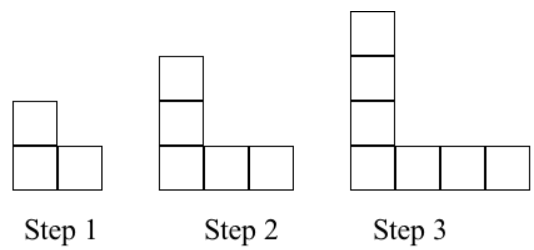
v. $F \cup M$

vi. $F \cap M'$

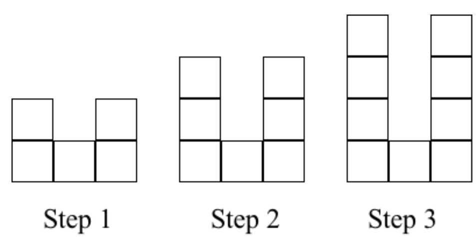
Grade 11-12: Sequences and Number Patterns

For each of the following:
a.) Draw Step 4.
b.) How many blocks are in Step 10?
c.) How many blocks are in Step *n*?

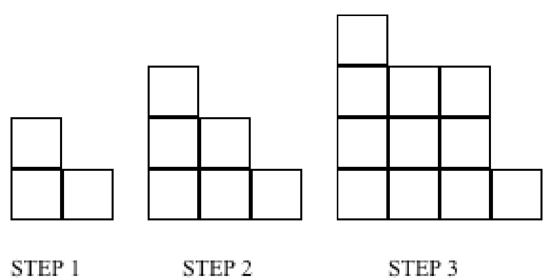
Problem 1.



Problem 2.



Problem 3.



Grade 11-12: Linear Programming

Linear programming involves reducing a set of **constraints** (conditions) to inequalities in two **variables**. Identifying the two variables is a critical first step. You then plot the inequalities on a graph. You will then have an area on the graph that shows all the points that satisfy all the conditions of the problem. This area is called the **feasible region**. The final consideration is a relationship between the two variables that needs to be maximized or minimized. This is called the **objective function**. By plotting this relationship on the graph and "sliding" this **search line** without changing its gradient you can find the maximum and minimum values, which will always occur at the vertices of the feasible area.

Be careful because many of the problems will need an integer number and the intersections of the inequalities are not always integers.

Step 1: Read the problem to identify the two unknown quantities, labeling them x and y.
Step 2: Write the inequality for each constraint.
Step 3: Graph the inequalities, identifying each boundary line and labeling the intersection points. All corners of the polygonal feasible region should be labeled. Shade the region.
Step 4: Express in terms of x and y the quantity that must be maximized or minimized. Solving the equation for y will identify the gradient of the "sliding" line.
Step 5: Draw the search line with the gradient from step 4, the one that first touches a corner of your polygon. Identify the coordinates of that point to determine the solution.

Example:

Production of Scooters and Tricycles

You have a supply of wheels, flats, rods and seats that you can use to make scooters and tricycles.

- Each scooter will need 2 wheels, 1 flat, 2 rods and 0 seats.
- Each tricycle will need 3 wheels, 1 flat, 4 rods and 1 seat.
- You have 30 wheels, 12 flats, 40 rods and 8 seats.

1. Can you make 8 scooters and 4 tricycles?
2. Can you make 4 scooters and 8 tricycles?
3. Mark the set of all points (x; y) in the plane, with $x \geq 0$, $y \geq 0$, for which you can make x scooters and y tricycles.
4. If you can sell each scooter for E100 and each tricycle for E200, how many of each should you make to maximize your total income?

Directed Numbers

Activity 1 – Addition and Subtraction

-10	- 9	- 8	- 7	- 6	- 5	- 4	- 3	- 2	- 1	0	1	2	3	4	5	6	7	8	9	10

Activity 2 – Multiplication

X	1	2	3	4
1				
2				
3				
4				

X	1	2	3	4
-4				
-3				
-2				
-1				
0				
1				
2				
3				
4				

x					0				
0									

Friend/Enemy “Reminder”
+ number = friend
- number = enemy

Algebra Word Problems

1. For the first 31 days of your new job, your boss offers you two salary options. The first option pays you R10 000 on the first day, R20 000 on the second day, R30 000 on the third day, and so on – in other words, R10 000*n* on the *nth* day. The second option pays you 10c on the first day, 20c on the second day, 40c on the third day – the amount doubling from one day to the next. Which option do you prefer, and why?

2. Pick any number. Add 4 to it and then double your answer. Now subtract 6 from that result and divide your new answer by 2. Write down your answer. Repeat these steps with another number. Continue with a few more numbers, comparing your final answer with your original number. Is there a pattern to your answers?

3. Guessing birthdays. Pat is working a number trick on Kim, whose birthday is the 29th of February. The table below shows the sequence of questions that Pat asks, as well as the calculations that Kim makes in response. Another column is provided for the algebra you are going to do to solve the trick. Use the letters m and d for month and day.

Instruction	*Kim*	*Algebra*
Write the number of your birthmonth	2	m
Multiply by 5	10	
Add 7	17	
Multiply by 4	68	
Add 13	81	
Multiply by 5	405	
Add the day of the month of your birthday	434	

After hearing the result of the last calculation, Pat can do a simple mental calculation and then state Kim's birthday. Explain how. To test your understanding of this trick, try it on someone whose birthday is unknown to you.

4. There are 396 persons in a theater. If the ratio of women to men is 2:3, and the ratio of men to children is 1:2, how many men are in the theater?

5. At noon one day, the river peaked at 11 feet above flood stage. It then began to recede, its depth dropping at 4 inches per hour.
(a) At 3:30 that afternoon, how many inches above flood stage was the river?
(b) Let *t* stand for the number of hours since noon, and h stand for the corresponding number of inches that the river was above flood stage. Make a table of values, and write an equation that expresses h in terms of t.
(c) Plot *h* versus *t*, putting *t* on the horizontal axis.
(d) For how many hours past noon was the river at least 36 inches above flood stage?

6. Samantha and Kate are solving the inequality $132 - 4x < 36$. Each begins by subtracting 132 from both sides to get $-4x < -96$, and then each divides both sides by -4. Samantha gets $x < 24$ and Kate gets $x > 24$. Always happy to offer advice, Alex now suggests to Samantha and Kate that answers to inequalities can often be checked by substituting x = 0 into both the original inequality and the answer. What do you think of this advice? Graph each of these answers on a number line. How do the results of this question relate to the flooding of the river?

7. Consider the sequence of numbers 2, 5, 8, 11, 14,..., in which each number is three more than its predecessor.
   (a) Find the next three numbers in the sequence.
   (b) Find the 100^{th} number in the sequence.
   (c) Using the variable n to represent the position of a number in the sequence, write an expression that allows you to calculate the nth number. The 200^{th} number in the sequence is 599. Verify that your expression works by evaluating it with n equal to 200.

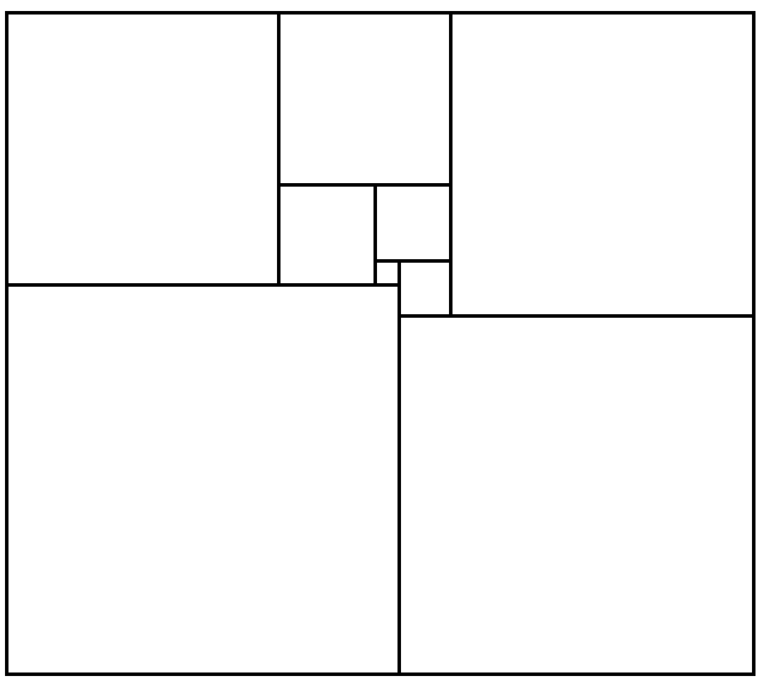
8. Several students were meeting in a room. After 45 of them left, the room was 5/8 as full as it was initially. How many students were in the room at the start of the meeting? The sum of four consecutive integers is 2174. What are the integers?

9. Each beat of your heart pumps approximately 0.06 liter of blood.
   a) If your heart beats 50 times, how much blood is pumped?
   b) How many beats does it take for your heart to pump 0.48 liters?

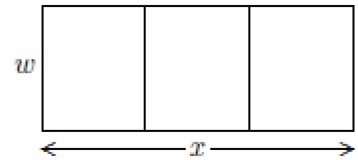
10. (Continuation) Direct-variation equations can be written in the form $y = kx$, and it is customary to say that y depends on x. Find an equation that shows how the volume V pumped depends on the number of beats n. Graph this equation, using an appropriate scale, and calculate its slope. What does the slope represent in this context?

11. A toy manufacturer is going to produce a new toy car. Each one costs $3 to make, and the company will also have to spend $200 to set up the machinery to make them.
   a) What will it cost to produce the first hundred cars? The first n cars?
   b) The company sells the cars for $4 each. Thus the company takes in $400 by selling one hundred cars. How much money does the company take in by selling n cars? How many cars does the company need to make and sell in order to make a profit?

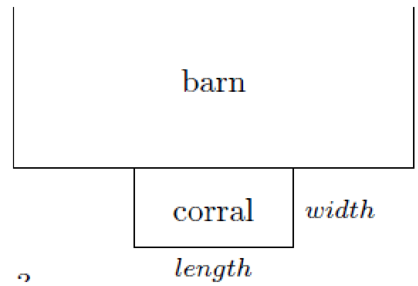
12. The diagram at right shows a rectangle that has been cut into nine square pieces, no two being the same size. Given that the smallest piece is 2 cm by 2 cm and using a variable, figure out the sizes of the other eight pieces. You will discover the hidden equation.



13. A farmer has 90 meters of fencing material with which to construct three rectangular pens side-by-side as shown at right. If w were 10 meters, what would the length x be? Find a general formula that expresses x in terms of w.



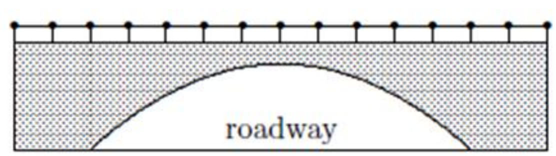
14. Sam breeds horses, and is planning to construct a rectangular corral next to the barn, using a side of the barn as one side of the corral. Sam has 240 feet of fencing available, and has to decide how much of it to allocate to the width of the corral.
(a) Suppose the width is 50 feet. What is the length? How much area would this corral enclose?
(b) Suppose the width is 80 feet. What is the enclosed area?
(c) Suppose the width is *x* feet. Express the length and the enclosed area in terms of *x*.



15. (Continuation) Let *y* stand for the area of the corral that corresponds to width *x*. Notice that *y* is a quadratic function of *x*. Sketch a graph of *y* versus *x*. For what values of *x* does this graph make sense? For what value of *x* does *y* attain its largest value? What are the dimensions of the corresponding corral?

16. In performing a controlled experiment with fruit flies, Wes finds that the population of male fruit flies is modeled by the equation $m = 2.2t^2 - 1.6t + 8$, while the female population is modeled by the equation
$f = 1.6t^2 + 2.8t + 9$, where *t* is the number of days since **the beginning of the first day** (thus $t = 2$ is the end of the second day). Assume that all flies live for the duration of the experiment.
(a) At the *beginning* of the first day, there are how many more female flies than male flies?
(b) Do male flies ever outnumber female flies? If so, when does that occur?
(c) Find an equation that models the total number *n* of flies that exist at time *t*. How many are present at the end of the tenth day? At what time are there 1000 fruit flies in the population?

17. The figure shows a bridge arching over the Laconic Parkway. To accommodate the road beneath, the arch is 100 feet wide at its base, 20 feet high in the center, and parabolic in shape.



(a) The arch can be described by $y = ax(x - 100)$, if the origin is placed at the left end of the arch. Find the value of the coefficient *a* that makes the equation fit the arch.
(b) Is it possible to move a rectangular object that is 40 feet wide and 16.5 feet high (a wide trailer, for example) through the opening? Explain.

18. Coffee beans lose 12.5% of their weight during roasting. In order to obtain 252 kg of roasted coffee beans, how many kg of unroasted beans must be used?

19. You have deposited $1000 in a money-market account that earns 8 percent annual interest. Assuming no withdrawals or additional deposits are made, calculate how much money will be in the account one year later; two years later; three years later; t years later.

20. The population of Grand Fenwick has been increasing at the rate of 2.4 percent per year. It has just reached 5280 (a milestone). What will the population be after ten years? after t years? After how many years will the population be 10560?

21. A helium-filled balloon is slowly deflating. During any 24-hour period, it loses 5 percent of the helium it had at the beginning of that period. The balloon held 8000 cc of helium at noon on Monday. How much helium did it contain 3 days later? 4.5 days later? 20 days later? n days later? 12 hours later? k hours later? Approximately how much time is needed for the balloon to lose half its helium? This time is called the *half-life*. Be as accurate as you can.

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