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basic education

Department: Basic Education REPUBLIC OF SOUTH AFRICA

Southern Africa

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MATHS
PORT ELIZABETH
JULY 4 - 8, 2016

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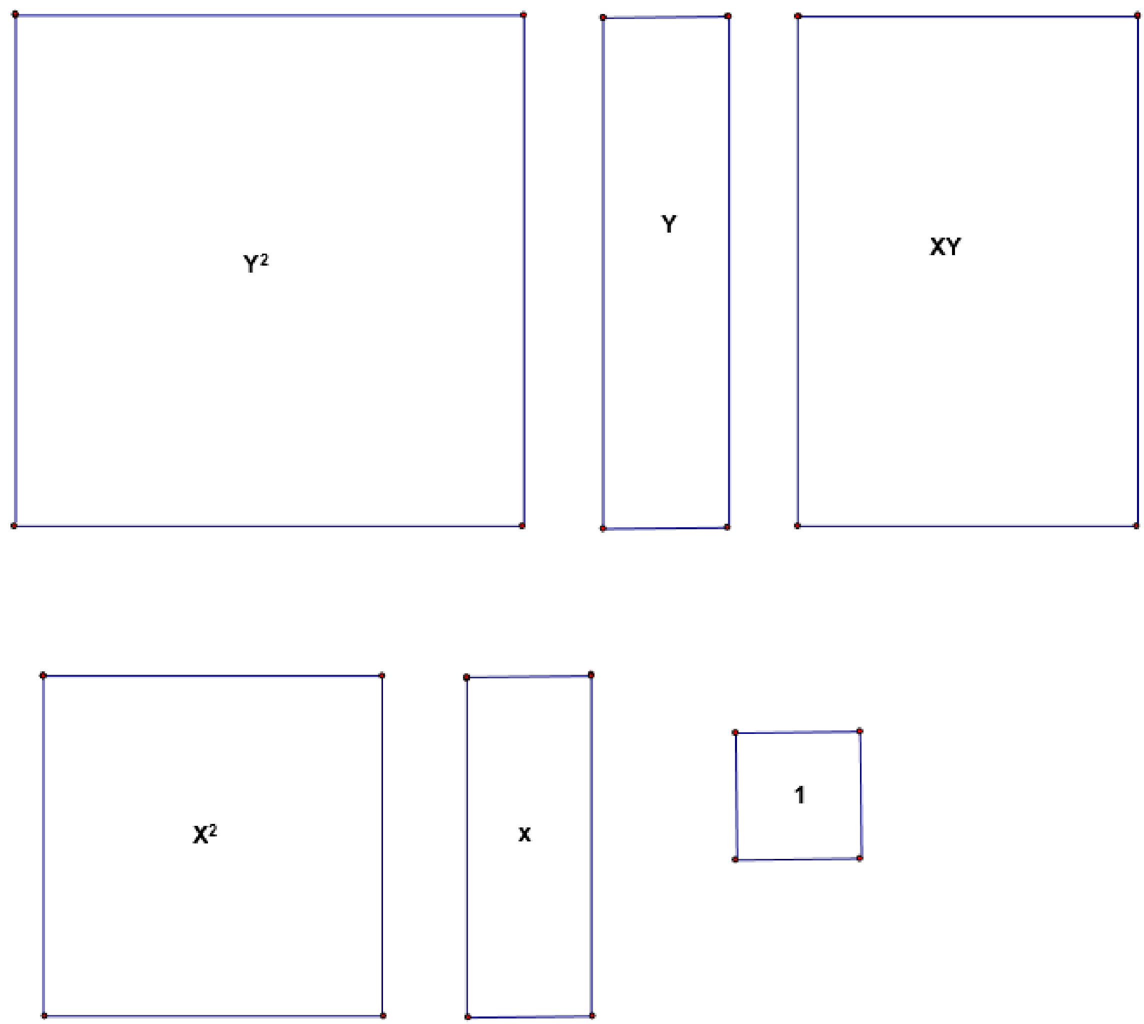
Grades 7-9 Priority Topics: Algebraic Expressions and Problem Solving

Algebraic Expressions

Algebra Blocks



1. A rectangle whose length is x and whose width is 1 is called an x-*block*. Which of the figures above is an x-block?

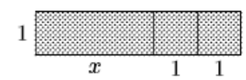
    (a) What is the area of an x-block?

    (b) What is the combined area of two x-blocks?

    (c) Show that there are two different ways to combine two x-blocks to form a rectangle whose area is $2x$.

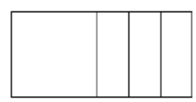
    (d) Use your algebra blocks to create two different rectangular diagrams to show that $x + 2x = 3x$.

2. There are several more algebra blocks. The 1-by-1 square is called a *unit block* or a *1-block*. As shown below, we can represent $x + 2$ by placing together an x-block and two 1-blocks. Use an appropriate number of x-blocks and 1-blocks to illustrate the distributive property $3(x + 2) = 3x + 6$.



3. The x^2-*block* above is another member of the algebra-block family. Use your algebra blocks to show that $x(x + 2) = x^2 + 2x$.

4. Label the figure below so that it provides a geometric representation of $x(x+3)$. Notice that this question is about *area*.



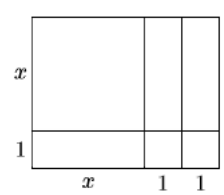
5. The y-*block* and xy-*block* are two more members of the algebra-block family. Use your algebra blocks to illustrate the equation $(1 + x)y = y + xy$.

6. The last member of the algebra-block family we have is y^2-*block*. Show how an xy-block and a y^2-block can be combined to illustrate the equation $(x + y)y = xy + y^2$.

7. Multiply $2 + x$ by $2x$. Use algebra-blocks to illustrate this calculation.

8. Multiply $x + 2y$ by $3y$. Use algebra-blocks to illustrate this calculation.

9. Write $(x + 1)(x + 2)$ without parentheses. Explain how the diagram below illustrates this product.



10. Create rectangles that are composed of x^2-blocks, x-blocks, and 1-blocks to illustrate the results when the following *binomial* products are expanded:
(a) $(x + 2)(x + 3)$ (b) $(2x + 1)(x + 1)$ (c) $(x + 2)(x + 2)$

11. By rearranging the pieces of the puzzle shown below, you can demonstrate that $x^2 - 4$ is equivalent to $(x + 2)(x - 2)$ *without* using the distributive property. Show how to do it.

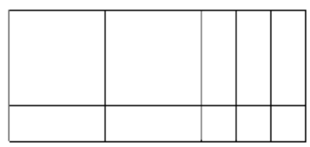
12. Create a rectangle using two x^2-blocks and two x-blocks. Write the dimensions of your rectangle. What is the area of the rectangle?

13. (Continuation) Using the same two x^2-blocks and the same two x-blocks, create a different rectangle. What is the area of the rectangle.

14. (Continuation) One of your diagrams illustrates the equation $x(2x + 2) = 2x^2 + 2x$. Explain. Write an equation that is illustrated by the other diagram.

15. Create rectangles that are composed of x^2-blocks, x-blocks and 1-blocks to illustrate the results when the following binomials are expanded:
(a) $(x + 1)(x + 1)$ (b) $(x + 2)(x + 2)$ (c) $(x + 5)(x + 5)$
It is common to write $(x + n)(x + n) = (x + n)^2$.

16. The diagram below consists of two x^2-blocks, five x-blocks, and three 1-blocks. Use this diagram to write a statement that says the product of the length and width of this particular rectangle is the same as its area. Can you draw another rectangle with the same area but different dimensions?



17. Create a rectangle by combining two x^2-blocks, three x-blocks and a single 1-block. Write expressions for the length and width of your rectangle. Using these expressions, write a statement that says that the product of the length and width equals the area.

18. (Continuation) Instead of saying, "Find the dimensions of a rectangle made with two x^2-blocks, three x-blocks and one 1-block", mathematicians say "*Factorise* $2x^2+3x+1$." It is also customary to write the answer $2x^2 + 3x + 1 = (2x + 1)(x + 1)$. Explain why the statement about the blocks is the same as the algebraic equation.

19. Factorise each expression and draw an algebra-block diagram:
(a) $3x^2 + 12x$ (b)$x^2 + 5x + 6$ (c) $4xy + 2y^2$

20. You have one x^2-block, six x-blocks (all of which you must use), and a supply of 1-blocks. How many different rectangles can you make? Draw an algebra-block diagram for each.

21. Take an x^2-block and 2 x-blocks. Show how you can make a square with these blocks if you add one 1-block. What are the length and width of the square you built? Complete the following. The blank on the right represents the length of the side of the square you made.
$$x^2 + 2x + \_\_\_\_ = x^2 + 2x + (\_\_\_\_)^2 = (\_\_\_\_\_\_\_\_)^2$$

22. Take an x^2-block and 6 x-blocks. How many 1-blocks must you add to make a square? What are the length and width of the square you built? Complete the following. The blank on the right represents the length of the side of the square you made.
$$x^2 + 6x + \_\_\_\_ = x^2 + 6x + (\_\_\_\_)^2 = (\_\_\_\_\_\_\_\_)^2$$

Problem Solving

Juniper Green

This game was described by Ian Stewart in his Mathematical Recreations column in the March 1997 issue of Scientific American, page 118. Juniper Green is the name of the school where it was first played. It was originally designed to teach multiplication and division. Students gain familiarity with multiples and factors through this game while engaging their problem solving skills as they attempt to find a winning strategy.
Materials:

- Card paper cut into rectangles with a number from 1 to 100.
- OR A large 10 x 10 chart with the numbers 1 through 100 written and counters or markers to cover each number as it is chosen.

The Rules:

1. Two players take turns removing a card (or covering a number). Cards removed are not replaced and not used again.
2. The first card chosen must be even.
3. Apart from the first move each card taken must be an exact multiple of the previous card or an exact divisor of the previous card.
4. The first card taken must be an even number.
5. The first player unable to take a card loses.

Extension:
Call JG-n a game of Juniper Green with cards 1 through n. For which values of n from 2 through 10 in JG-n does the first player have a winning strategy?

Number puzzles

1. Choose any number. Double it. Subtract six and add the original number. Now divide by three. Repeat this process with other numbers until a pattern develops. By using a *variable* such as x in place of your number, show that the pattern does not depend on which number you choose initially.
2. Pick a number, add 5 and multiply the result by 4. Add another 5 and multiply the result by 4 again. Subtract 100 from your result and divide your answer by 8. How does your answer compare to the original number? You may need to do a couple of examples like this until you see a pattern.
3. Consider the sequence of numbers 2, 5, 8, 11, 14, ...
   (a) Find the next three numbers in the sequence.
   (b) Find the 100th number in the sequence.

(c) Using the variable n to represent the position of a number in the sequence, write an expression that allows you to calculate the nth number. The 200th number in the sequence is 599. Verify that your expression works by evaluating it with n equal to 200.

4. Pat is working a number trick on Kim, whose birthday is the 29th of February. He asks her to do the following sequence of computations: *Write the number of your birthmonth. Multiply by 5. Add 7. Multiply by 4. Add 13. Multiply by 5. Add the day of the month of your birthday.* Upon hearing 434, the result of Kim's last calculation, Pat can do a simple mental calculation and then state her birthday. Explain his method. To test your understanding of the trick, see if you can figure out my birthday if the result of my calculations is 921.

5. Imagine that we have a circular pizza and a knife. You are asked to make n straight cuts of the pizza so that the number of slices, s_n, is maximized. What is s_1, the maximum number of slices possible after one cut? s_2, the maximum number of slices possible after two cuts? s_3, the maximum number of slices possible after three cuts? What about s_4? s_5? Do you notice a pattern to your answers? Can you describe how to find s_n from the previous answer?

6. An elf has a staircase to climb. Each step it takes can cover either one stair or two stairs. In how many ways can the elf climb 1 stair? What about 2 stairs? 3 stairs? 4 stairs? 5 stairs? Do you notice a pattern? Can you describe the number of different ways the elf can climb n stairs, in terms of the previous answers?

More puzzles

7. A school has 1000 numbered lockers, one for each of the 1000 students. One day, students performed a curious exercise. The first student went in and opened all of the lockers. A second student then went and closed every other locker, leaving the first one open. A third student went and altered the state of every third locker, leaving two lockers open then switching the state of the next - if it were open, the student closed it, and if it were closed, the student opened it. In turn, the fourth student altered the state of every fourth locker. This routine continues until, finally, the thousandth student went in an altered the 1000^{th} locker's state. How many lockers were left open in the end, and which ones were they?

8. A local tennis club as a single elimination tournament. The 128 players are paired to form the first day's matches. The winners of these matches are paired to form the second day's matches, etc. How many days does the tournament last? How many matches are played in all?

9. Twelve business associates meet for lunch. As they leave to return to their offices a couple of hours later, one of them conducts a small mathematical experiment, asking each one in the group how many times he or she shook hands with someone else in the group. The twelve reported values were 3, 5, 6, 4, 7, 5, 4, 6, 5, 8, 4, and 6. What do you think of these data? Are their answers believable?

10. Mr. Mbuto and his wife recently attended a party at which there were four other married couples. Various handshakes took place. No one shook hands with their own spouse, and no two persons shook hands more than once. After all the handshakes were over, he asked each person, including his wife, how many times he/she had shaken hands. To his surprise, each person gave a different answer. How many times did Mr. Mbuto's wife shake hands?

Polya's Problem Solving Principles

1. First, you have to understand the problem.
2. After understanding, make a plan.
3. Carry out the plan.
4. Look back at your work. How could it be better?

First Principle: Understand the Problem
Some questions to ask while trying to understand the problem:

- What are you asked to find or show?
- Can you restate the problem in your own words?
- Can you think of a picture or a diagram that might help you understand the problem?
- Is there enough information to enable you to find a solution?
- Do you understand all the words used in stating the problem?

Second Principle: Devise a Plan
There are many reasonable ways to solve problems. The skill at choosing an appropriate strategy is best learned by solving many problems. You will find choosing a strategy increasingly easy. A partial list of strategies is below:

- Guess and check
- Make an orderly list
- Eliminate possibilities
- Use symmetry
- Consider special cases
- Use direct reasoning
- Solve an equation
- Look for a pattern
- Draw a picture
- Solve a simpler problem
- Use a model
- Work backward
- Use a formula
- Be creative
- Use your head/noggin

Grades 10-12 Priority Topics: Algebra and Financial Math

Algebra

Exponents

1. Faced with the problem of multiplying 5^6 by 5^3, Maya is having trouble deciding which of these four answers is correct: 5^{18}, 5^9, 25^{18}, or 25^9. Your assistance is needed. Once you have answered Maya's question, experiment with other examples of this type until you can formulate the *common-base principle for multiplication* of exponential expressions.

2. Faced with the problem of calculating $(5^4)^3$, Maya is having trouble deciding which of these three answers is correct: 5^{64}, 5^{12} or 5^7. Once you have answered Maya's question, experiment with other examples of this type until you can formulate the principle that applies when exponential expressions are raised to powers.

3. Maya now needs to divide 5^{24} by 5^8 and is having trouble deciding which of the following four answers is correct: 5^{16}, 5^3, 1^16, or 1^3. Can you help her with this problem? Once you have, experiment with other examples of this type until you can formulate the *common-base principle for division* of exponential expressions.

4. (Continuation) Apply the common-base principle for division to the following situations:

   (a) earth's human population is roughly 6×10^9, and its total land area excluding the polar caps is roughly 8×10^7 square kilometres. If the human population were distributed uniformly over all available land, approximately how many persons would be found per square kilometre?

   (b) At the speed of light, which is 3×10^8 meters per second, how many seconds does it take the sun's light to travel the 1.5×10^{11} meters to earth?

5. The common-base principle for multiplication predicts that $5^{1/2}$ times $5^{1/2}$ should be 5. Explain this logic, then conclude that $5^{1/2}$ is just another name for a familiar number. How would you describe the number $6^{1/3}$ given that $6^{1/3} \times 6^{1/3} \times 6^{1/3}$ equals 6? Formulate a general meaning of expressions like $b^{1/n}$ and test your interpretation on simple examples such as $8^{1/3}$.

Logarithms

6. Write each of the following numbers as a power of 10.

   (a) 1000

   (b) 1000000

   (c) 0,01

   (d) $\sqrt{10}$

   (e) $100\sqrt{10}$

   (f) $\dfrac{1}{\sqrt[3]{10}}$

7. Given a positive number p, the solution to $10^x = p$ is called the *base-10 logarithm of* p, expressed as $x = \log_{10} p$, or simply $x = \log p$. For example $10^4 = 10000$ means that 4 is the base-10 logarithm of 10000 or $4 = \log 10000$. Remembering that *logarithms are exponents*, explain why it is predictable that
   (a) $\log 64$ is three times $\log 4$
   (b) $\log 12$ is the sum of $\log 3$ and $\log 4$
   (c) $\log 0,02$ and $\log 50$ differ only in sign/direction.
8. Given that $m = \log a$, $n = \log b$ and $k = \log(ab)$,
   (a) express a, b, and ab as powers of 10;
   (b) use your knowledge of exponents to discover a relationship among m, n, and k;
   (c) conclude that $\log(ab) = \log a + \log b$. This is the *product rule for logarithms.*
9. (Continuation) Justify the rules:
   (a) $\log a^r = r \log a$. This is the *power rule for logarithms.*
   (b) $\log(a/b) = \log a - \log b$. This is the *quotient rule for logarithms.*
10. Solve each of the following equations:
    (a) $8^x = 32$
    (b) $27^x = 243$
    (c) $1000^x = 100000$
11. (Continuation) Given a positive numbers b and p, the solution to $b^x = p$ is called the *base-b logarithm of* p, expressed as $x = \log_b p$. Write the exponential equations above as a logarithmic equation.
12. (Continuation) In the previous problem, you solved for x in the equation $8^x = 32$ by using a logarithm with a base other than 10. Here, Thokozani solved for x using base-10 logarithms. Justify each step:

$$
\begin{aligned}
8^x &= 32 \\
\log 8^x &= \log 32 \\
x \log 8 &= \log 32 \\
x &= \frac{\log 32}{\log 8}
\end{aligned}
$$

13. (Continuation) Use reasoning similar to Thokozani's above to solve $27^x = 243$ for x using base-10 logarithms.
14. Justify the *change-of-base formula*, which says $\log_b p = \dfrac{\log p}{\log b}$, by solving the equation $b^x = p$ in two ways.

Finance, Growth and Decay

1. On 1 July 2012, you deposit 1000 rand into an account that pays 6% interest annually. How much is this investment worth on 1 July 2032?

2. (Continuation) On 1 July 2013, you deposit 1000 rand into an account that pays 6% interest annually. How much is *this* investment worth on 1 July 2032? Answer the same question for 1 July 2014, 1 July 2015, and so forth, until you see a pattern developing in your expressions.

3. (Continuation) Suppose that you deposit 1000 rand into the same account on 1 July *every* year. The problem now is to calculate the combined value of *all* these deposits on 1 July 2032, including the deposit made on the final day. Rather than getting the answer by tediously adding the results of *twenty-one* separate (but similar) calculations, we can find a shorter way. Let V stand for the number we seek, and observe that

$$V = 1000(1.06)^0 + 1000(1.06)^1 + 1000(1.06)^2 + \cdots + 1000(1.06)^{19} + 1000(1.06)^{20}$$

is the very calculation we wish to avoid. Obtain a second equation by multiplying both sides of this equation by 1.06, then find a way of combining the two equations to obtain a compact, easy-to-calculate formula for V.

4. (Continuation) Any list *first*, *first* $\times$ *multiplier*, *first* $\times$ *multiplier*^{2}, ..., in which each term is obtained by multiplying its predecessor by a fixed number is called a *geometric sequence*. A *geometric series*, on the other hand, is an addition problem formed by taking consecutive terms from some geometric sequence. For example, $32 - 16 + 8 - 4 + \cdots + 0,125$ is a nine-term geometric series whose sum is 21,375. Consider now the typical geometric series, which looks like *first*+*first* $\times$ *multiplier*+*first* $\times$ *multiplier*^{2}+ ... +*last*. Find a compact, easy-to-calculate formula for the sum of all these terms.

5. Sometimes it is necessary to invest a certain amount of money at a fixed interest rate for a fixed number of years so that a financial goal is met. The initial amount invested is called the *present value* and the goal is called the *future value*. The parents of an American child decide that \$350 000 will be needed for college expenses. They find a certificate of deposit that pays 0,5 percent interest each month. How much (present value) should they invest so that there is \$350 000 on the child's 18th birthday?

6. (Continuation) The parents realize they do not have that type of cash, so they decide they will deposit the same amount every month into the account, with their last payment on the child's 18th birthday. How much must they invest per month in order to have \$350 000 in the account on the child's 18th birthday?

7. Suppose that you invest 10000 rand in an account that pays 0,5% interest per month. In how many months will the account have double your initial investment?

8. Nthoko invests 10000 rand in a stock that returned 0,5% monthly interest for 2 years. Unfortunately, that stock took a nose dive and the value fell by 0,5% each month for two years. How much money did he have after the 4 years?

9. *Repaying Loans.* A bank has just granted Jordan a 10 000 rand loan, which will be paid back in 48 equal monthly installments, each of which includes a 1% interest charge on the unpaid balance. The loan officer was amazed that Jordan (who knows about geometric series) had already calculated the monthly payment. Here's how Jordan figured it out:

   (a) Pretend first that the monthly payments are all 300R. The first payment must include 100R just for *interest* on the 10 000R owed. The other 200R *reduces the debt*. That leaves a debt of 9800R after the first payment. Follow this line of reasoning and calculate the amount owed after four more payments of 300R have been made.

   (b) Some notational shorthand: Let A_n be the amount owed after n payments (so that $A_0 = 10000$), let $i = 0,01$ be the monthly interest rate, and let M be the monthly payment (which might not be 300R).

      i. Explain why $A_1 = A_0 - (M - iA_0) = (1+i)A_0 - M$.
      ii. Explain why $A_2 = (1+i)A_1 - M$.
      iii. Write an equation for A_n, the amount owed after n payments, in terms of A_{n-1}, the amount owed after one fewer payment.

   (c) We wish to write A_n in terms of A_0. For example, you could write

$$\begin{aligned} A_2 &= (1+i)A_1 - M \\ &= (1+i)[(1+i)A_0 - M] - M \\ &= (1+i)^2 A_0 - (1+i)M - M \end{aligned}$$

      i. How could we write A_3 in terms of A_0? You should see a pattern developing!
      ii. How could we write A_n in terms of A_0? Your answer should involve the finite geometric series $M + (1+i)M + (1+i)^2 M + \cdots + (1+i)^{n-1}M$.
      iii. Simplify your answer for the previous part by writing the finite geometric series in a more compact form.

   (d) If N is the total number of monthly payments made, explain why $A_N = 0$ for Jordan's loan. Explain why $M = \dfrac{iA_0(1+i)^N}{(1+i)^N - 1}$.

   (e) Calculate Jordan's monthly payment using the formula from (d). In Jordan's case, the monthly payment is less that 300R.

10. What is the monthly payment needed to repay a 500000R loan in 10 years, if the bank charges 0,8 percent per month?

11. In December, Natasha took out a 10000R loan with monthly interest rate 0,7 percent. In order to pay back the loan, Natasha has been paying 871,70 a month since January. How many payments does Natasha need to pay everything back? How much does this loan actually cost her? What would the monthly payment have been if Natasha had been scheduled to pay back the loan in 24 months?

Prelude: Workshop focus considerations brad.uy@gmail.com
Grades 7-9: Geometry (Congruence and Similarity; Informal Proofs)
Congruence and Similarity

I. Definitions of congruence and similarity, considering parts of 2-D figures (angles and sides)
   A) Figures are congruent when …
   B) Figures are similar when…
      1. topic of proportionality, illustrating it...
      2. scale factor vs scale ratio
   C) Informal definitions using terms shape and size

- Learners should recognize that two or more figures are congruent if they are equal in all respects i.e. angles and sides are equal.
- Learners should recognize that two or more figures are similar if they have the same shape, but differ in size i.e. angles are the same, but sides are proportionally longer or shorter. Similar figures are further explored when doing enlargements and reductions. Refer to "Clarification Notes" under 3.4 Transformation Geometry.

II. Lab / demo / illustration of congruence and similarity, helping us to understand and recognize difference in concepts
   A) Flashlights and plastic or glass, or cutouts
   B) Can images be rotated or flipped and still preserve similarity or congruence?

C1 True or false? If two rhombuses have sides which are proportional (say, with a scale ratio of 1:2), then the two rhombuses are similar.
Discuss...
Related query: true or false? If the sides of two figures are proportional, then they are similar.
T or F? If two figures are similar, then they are congruent.
If two figures are congruent, then they are similar.

- Comparing rhombii with sides proportional, learners can ascertain that having sides proportional does not necessarily imply that the corresponding angles will be equal. So only having sides of proportional length is not a sufficient condition for similarity

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III. Exploring effects of enlargements (or reductions) of similar figures
   A) From Caps

Enlargements and reductions
- Draw enlargements and reductions of geometric figures on squared paper and compare them in terms of shape and size

Enlargements and reductions
- Use proportion to describe the effect of enlargement or reduction on area and perimeter of geometric figures
- Investigate the co-ordinates of the vertices of figures that have been enlarged or reduced by a given scale factor

   B) Proportions, commonly written as ratios, which we may refer to as scale ratio
   C) Effects on perimeter and area (and by extension, volume for 3-D shapes)

C2 Lab exploration: Let's start with a square. Given a square with side length of 10 units. Draw it. Calculate The perimeter.

	Scale factor	Perimeter	Original perimeter has been multiplied by ___? = (Perim of new figure) / (**Perim of original**) = Perimeter-change multiplier
image	1/2		
Original shape	**1**		
image	2		

image	3		
image	4		
image	5		

Let's say we enlarge this square by a factor of 2. What is the length of each side? Calculate the perimeter and compare it to the original.

Let's say we enlarge the original square by a factor of 3. What is the length of each side? Calculate the perimeter and compare it to the original.

What general, universal rule can we develop?

(If you enlarge a figure by a factor of ___, then the perimeter will change by a factor of ___….
If the scale factor is __, then the perimeter will change by a factor of __.)

Make educated guesses to fill in the rest of the table. Check your answers.

What is the perimeter of the imaged square if the scale factor is ½? Confirm your answer.

IV. Discussion of perimeters of similar figures

A) This exercise may be repeated for a rectangle of dimensions 2 units by 4 units.

B) Or an obtuse triangle with side lengths, 4, 6, and 8.

C) Note we haven't discussed proportions yet. But we can say that the proportion (ratio) of perimeters of similar shapes is equal to the proportion (ratio) of their corresponding sides (this is the scale ratio).

1. For example, let's say we have similar octagons. The scale ratio is 1:4. If the perimeter smaller octagon is 20 cm, what is the perimeter of the larger octagon.

V. Moving on to effect of enlargement on area

A) Question to motivate discussion: if we enlarge a figure by a factor of 2, will the area increase by a factor of 2?

B) Any guesses? Counterexamples?

C**3** Lab: Let's start with a square. Given a square with side length of 4 units. Draw it. Calculate the area.

	Scale factor	Area	Original area has been multiplied by ___? = (Area of new figure) / (**Area of original**) = Area-change multiplier
image	1/2		
Original shape	**1**		**1**
image	2		
image	3		
image	4		
image	5		

Let's say we enlarge this square by a factor of 2. What is the length of each side? Calculate the

area and compare it to the original.
Let's say we enlarge the original square by a factor of 3. What is the length of each side?
Calculate the area and compare it to the original.
What general, universal rule can we develop?

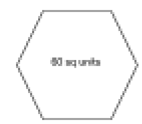
(If you enlarge a figure by a factor of ___, then the area will change by a factor of ___….
If the scale factor is __, then the area will change by a factor of __.)

Make educated guesses to fill in the rest of the table. Check your answers.
What is the area of the imaged square if the scale factor is ½? Confirm your answer.

--This exercise may be repeated for a rectangle of dimensions 2 units by 4 units.
--Or a right triangle with side lengths 6, 8, 10.

C4 Note we haven't discussed proportions yet. But we can say that the proportion (ratio) of areas of similar shapes is equal to the square of the proportion (ratio) of their corresponding sides (this is the scale ratio).
--For example, say we have two squares with a scale ratio of 1:4. The smaller square has an area of 8 sq. units. What is the area of the larger square?
--we can prove this by finding the side lengths of the smaller and larger squares....

C5 A hexagon has an area of 60 sq units.
What will the area of the new hexagon be if it is enlarged by a factor of 3?



VI. Prefiguring rules for similar figures and their volume
A) Explore...If we enlarge a cube by a factor of 2, will the volume increase by a factor of 2?
1. Test with numbers. Say start with a cube of side length 5 cm. Solve...
2. Generate a rule

VII. Dilations and coordinates of images??

Informal Proofs

I. Overview
A) Basis in postulates and definitions
B) Written in paragraph form with reasons and justifications for support
C) Strategy
1. Form a game plan before beginning
2. Use diagrams and markings

II. Common Postulates and definitions
A) Algebraic properties
1. Addition property of equality
2. Subtraction property of equality
3. Multiplication property of equality
4. Division property of equality
5. Substitution property of equality
6. Reflexive property of equality (or of congruence)
7. Transitive property of equality (or of congruence)
8. Simplification
B) Definitions
1. Definition of a midpoint (may be a theorem that two segments are equal)

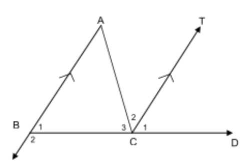
2. Definition of an angle bisector
3. Definition of a segment bisector
4. Definition of perpendicular lines
5. CPCTC (corresponding parts of congruent triangles are congruent)
    a) special case of definition of congruence
6. Definition of similarity

C) Postulates
1. Segment addition postulate
2. Angle addition postulate
3. The sum of the angles of a triangle is 180^0

D) Theorems
1. For proving triangles congruent
    a) SSS, ASA, SAS, AAS, and HL
        1. Note that AAA and ASS are not allowed….why
2. Given parallel lines cut by a transversal
    a) corresponding angles are congruent (postulate)
    b) alternate interior angles are congruent (try prove this)
    c) same side interior (co-interior) angles are supplementary
    d) given these conditions, these are also ways to prove lines are parallel
3. Vertical angles are congruent
4. If two angles form a linear pair, then they are supplementary
5. The measure of the exterior angle of a triangle is equal to the sum of the two remote interior angles. (try prove this)
6. Opposite angles of a parallelogram are congruent
7. In a triangle, angles opposite congruent sides are congruent (isosceles triangle theorem)
8. All radii of a circle are congruent (possibly by definition)
9. AAA similarity theorem for triangles

P1 From ANA 2014

9.2

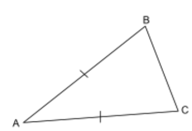


In the figure above, $AB \parallel TC$, $\hat{C}_1 = 65°$ and $\hat{C}_2 = 43°$. Calculate the size of $\hat{A}$, $\hat{B}_1$ and $\hat{B}_2$.

Statement	Reason

(3)

9.3

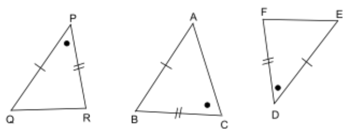


In ΔABC, $AB = AC$ and $\hat{C} = x°$. Determine the size of $\hat{A}$ in terms of x.

Statement	Reason

(3)

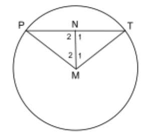
10.1



Which triangle is congruent to ΔPQR?

Statement	Reason

10.2 In the given figure, P and T are points on a circle with centre M. N is a point on a chord PT such that $MN \perp PT$.

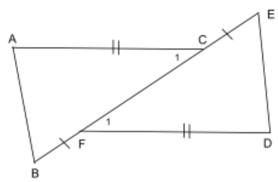


Prove that $PN = NT$.

Statement	Reason

(8)

10.3



In the above diagram, $AC = DF$, $AB = DE$ and $BF = CE$.

10.3.1 Prove that $BC = EF$.

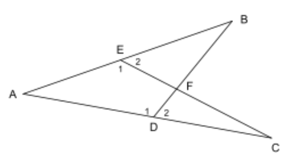
Statement	Reason

(2)

10.3.2 Prove that $\Delta ABC \equiv \Delta DEF$.

10.3.3 Why is angle B congruent to angle E? 10.3.4 What is the relationship between line AB and line ED?

10.4



In the figure, $\hat{B} = \hat{C}$, $AD = 9\ cm$, $AE = 7\ cm$ and $CE = 21\ cm$.

10.4.1 Prove that $\Delta ABD \parallel\!\parallel \Delta ACE$.

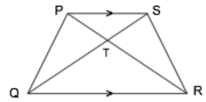
Statement	Reason

(6)

10.4.2 Calculate the length of BD.

P2 from ANA 2013

1.8 In the figure below, $PS \parallel QR$. Which ONE of the following statements is true for this figure?



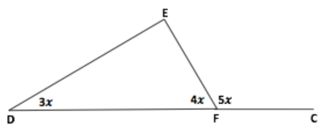
A ΔPTS ≡ ΔPQT

B ΔPTS ≡ ΔRTQ

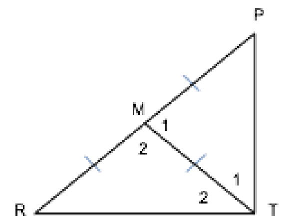
C ΔPTS ||| ΔSRT

D ΔPTS ||| ΔRTQ

1.9 In the figure below, side DF of ΔEDF is produced to C. Calculate the size of $\hat{E}$ in terms of x.



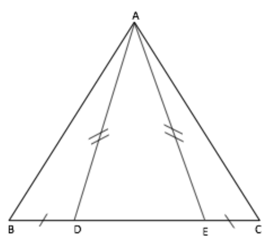
8.1 In ΔPRT below, M is the midpoint of PR and $MR = MT$.



If $\hat{P} = 25°$, calculate with reasons:

The measure of angle T1
The measure of angle M2
The measure of angle R

8.2 In ΔABC, D and E are points on BC such that $BD = EC$ and $AD = AE$.

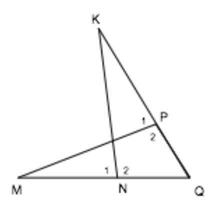


8.2.1 Why is $BE = CD$?

____ (1)

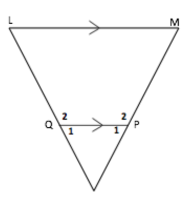
8.2.2 Which triangle is congruent to ΔABE ?

8.3 In the figure below ΔKNQ and ΔMPQ have a common vertex Q. P is a point on KQ and N is a point on MQ. $KQ = MQ$ and $PQ = QN$.



Prove with reasons that $\Delta KNQ \equiv \Delta MPQ$.

8.4 In ΔNML below, P and Q are points on the sides MN and LN respectively such that $QP \parallel LM$.
$MN = 16$ cm, $QP = 3$ cm and $LM = 8$ cm.



8.4.1 Complete the following (give reasons for the statements):
Prove with reasons that $\Delta QPN \ |||\ \Delta LMN$.

In ΔQPN and ΔLMN

1. $\hat{N}$ = ...
2. $\hat{P}_1$ = ...
3. $\hat{Q}_1$ = ...

∴ $\Delta QPN \ |||\ \Delta$......... .... (4)

8.4.2 Hence, calculate the length of PN.

Grades 10-12: Probability and Statistics

0. Birthday test
   - A) n / 365 ?

I. Foundations
   - A) represent a sample space
   - B) definition of probability

II. Counting methods
   - A) Fundamental counting principle, multiplying possibilities (with or without replacement)
   - B) Permutations (position) v. Combinations (committee) -- only used without replacement
   - C) Words with repetition

III. Complementarity method
   - A) Cues: the words _______________

1A Given 15 people. How many ways are there to create a basketball starting lineup of 5 people (positions matter)? How many ways are there to create a committee of 5 people? Now--how many different arrangements can you have if you elect one person as president, and have 4 serve as his unranked cabinet (hint: given a team/committee of 5 without assignments, how many different ways can you appoint the "captain?)?

Next weekend, you are expecting a visit from Lila and Emilio independently of one another. The probability that Lila will show up is .4, whereas the probability that Emilio will show up is .8. What is the probability that at least one of them will visit you next weekend?

1B When flipping 4 coins, what's the probability of getting at least one head?
When rolling five 12-sided dice, what's the probability of getting at least one three?

next transcribed from a matric November 2012

> Every client of CASHSAVE bank has a personal identity number (PIN) which is made up of 5 digits chosen from the digits 0 to 9. How many PINs can be made if digits can be repeated? If digits cannot be repeated? Suppose that a PIN can be made up by selecting digits at random and that the digits can be repeated. What is the probability that such a PIN will contain at least one 9?

1C How about some variation with letters? Prequestion: a passcode consists of a number followed by two lowercase letters. How many such passcodes exist in that specific order? Now, how many such passcodes exist if the the number can appear anywhere?

transcribed from november 2013 matric

> The nine letters of the word 'EQUATIONS' are used to form different five-letter codes. (1) How many different five-letter codes (without repetition) can be formed from the nine different letters in the word 'EQUATIONS'? (2) How many different five-letter codes can be formed from the letters in the word 'EQUATIONS' by using all the consonants and one vowel? Note--did you consider the pre-question in your answer here?

IV. Representations to assist in problem solving
   - A) Venn Diagrams
   - B) Tree Diagrams (follow scenario branches with multiplication)

V. Addition and Multiplication Rules (let's make a matrix here)
   - A) Addition rule general (or): plus for mutually exclusive and for independent events
     1. enact with large groups! avoid errors of double counting
   - B) Product rule general (and): plus for mutually exclusive and for independent events

C) Formal definition for independent v dependent events
1. "with replacement" and "without replacement" (always dependent)

VI. Conditional Probability (not explicitly examined, but still applicable)
A) Formula as manipulation of product rule

2A Given that P(M) = .2 and P (N) = .4 and P (M and N) = .1 , what is P (M or N) ? What is probability of one of M or N occurring, but not both? Are M and N mutually exclusive? Are they independent (harder question)? Restart: P(M) = .2 and P (N) = .4 . Is it possible that P (M or N) = .3 ? Hypothetically speaking (assuming we don't know if M and N are disjoint or not), what is the smallest P (M or N) can be? What is the largest value P (M or N) can assume?

2B transcribed from spring 2013 matric

The events A, B, and C are such: A and B are independent, B and C are independent and A and C are mutually exclusive. Their probabilities are P(A)= 0,3 , P(B)= 0,4 and P(C) = 0,2. Calculate the probability of the following events occurring: (1) Both A and C occur; (2) Both B and C occur; (3) At least one of A or B occur.

--19 envelopes each have 1 red paper in them, and 1 envelope has 19 blue papers in it. What is the probability of selecting one envelope at random and the paper being red? What's the moral here?

--Say in another drawer you have 15 envelopes which each have 5 red and 1 blue paper; and 5 envelopes each have have 2 red and 8 blue. What's the probability of picking one envelope and drawing out a paper and having it be red?

2C 1. If P(A) = .5 and P(B) = .8 , and P(A|B) = .4 , find P(B|A).
2. If P(A) = .25 and P(B) = .30 , and P(A|B) = .4 , find P(B|A).
Try to visualize these using Venn diagrams instead of using formulas…

from CAPS: In a survey 80 people were questioned to find out how many read newspaper S or D or both. The survey revealed that 45 read D, 30 read S and 10 read neither. Use a Venn diagram to find out how many read 1) S only. 2) D only. And 3) both D and S. *Demonstrations of these principles are easy with big groups--say, see how many people enjoy rice / vegetables / both. We can learn the importance of not double counting the intersection.*

2D transcribed from spring 2013 matric, question 6

A survey is conducted among 174 students. The results are shown below. 37 study Life Sciences; 60 study Physical Sciences; 111 study Mathematics; 29 study Life Sciences and Mathematics; 50 study Mathematics and Physical Sciences; 13 Study Physical Sciences and Life Sciences; 45 do not study any of Life Sciences, Mathematics, or Physical Sciences. x students study Life Sciences, Mathematics, and Physical Sciences. Draw a Venn diagram to represent the information above. Show that x = 13. If a student were selected at random, calculate the probability that he studies the following: (1) Mathematics and PHysical Sciences but not Life Sciences. (2) Only one of Mathematics or Physical Sciences or Life Sciences.

3A 4 friends (a, b, c, and d) each choose a random number between 1 and 5. What is the chance that any of them use the same number? or...Assuming that it is equally likely to be born in any of the 12 months of the year, what is the probability that in a group of six individuals, at least two people have birthdays in the same month? ….or, A person has three pairs of socks. If the person decides to pair the six socks in a random fashion, what is the probability that each sock has been properly matched? Reconsidering the first question--how many friends do you need picking a number between 1-5 to guarantee at least two people have the same number? Is this confirmed in the calculations?

3B transcribed from Spring 2013 matric

Consider the word: PRODUCT. How many different arrangements are possible if all the letters are used? How many different arrangements can be made if the first letter is T and the fifth letter is C? How many different arrangements can be made if the letters R, O, and D must follow each other, in any order [as in, they have to be adjacent, though can appear in any order. Hint: tiles?]?

3C spring 2012 matric

QUESTION 7

Three items from four different departments of a major chain store will be featured in a one-page newspaper advertisement. The page layout for the advertisement is shown in the diagram below where one item will be placed in each block.

A	B	C
D	E	F
G	H	I
J	K	L

(1) In how many different ways can all these items be arranged in the advertisement?
(2) In how many different ways can these items be arranged if specific items are to be placed in blocks A, F, and J?
(3) In how many different ways can these items be arranged in the advertisement if items from the same department are grouped together in the same row?

3D If you deliver 5 personal notes to 5 students' mailboxes randomly, what is the probability that every letter goes to the proper recipient?

Statistics (5 Number Summary interpretation, box and whisker diagram; ogives; scatter plots; linear regression line; correlation coefficient)

I. Data
  A) Classifying
    1. Univariate v bivariate data
      a) for univariate: we can use 5 number summary, box and whisker diagrams
        1. And ogives to represent distribution
      b) for bivariate data, we can use scatter plots and linear regression lines

II. 5 number summary and box and whisker diagrams--interpreting--think of halves and halves of halves
  A) use the summary to construct the diagram: min, max, Q1, Q2 (aka the median), and Q3
  B) rules for finding the quartiles

III. Ogives, give cumulative frequency--the number of occurrences **below** a certain amount (typically)
  A) Frequency distribution and cumulative frequency distribution
    1. rendered graphically with histograms and ogives

IV. Bivariate data using scatter plots
  A) correlation coefficient (as a measure of validity of below)
  B) linear regression formula, for the use of …

S1 In a grid, a, b, c, d, e, f, and g represent values in a data set in increasing order. No value is repeated. Determine the value of all the letters if the maximum value is 42, the range is 35, the median is 23, the difference between the median and the upper quartile is 14, the interquartile range is 22, e=2c, and the mean is 25.

S2

1. Revise symmetric and skewed data.
2. Use statistical summaries, scatterplots, regression (in particular the least squares regression line) and correlation to analyse and make meaningful comments on the context associated with given bivariate data, including interpolation, extrapolation and discussions on skewness.

Example:

The following table summarises the number of revolutions x (per minute) and the corresponding power output y (horse power) of a Diesel engine:

x	400	500	600	700	750
y	580	1030	1420	1880	2100

1. Find the least squares regression line $y = a + bx$ (K)
2. Use this line to estimate the power output when the engine runs at 800m. (R)
3. Roughly how fast is the engine running when it has an output of 1200 horse power? (R)

calculate corresponding linear correlation coefficient as well. Any efficient methods to share…

S3 transcribed from spring 2013 matric

The average percentage of 150 learners for all their subjects is summarised in the cumulative frequency table below.

PERCENTAGE INTERVAL	CUMULATIVE FREQUENCY
$x \le 10$	5
$x \le 20$	21
$x \le 30$	50
$x \le 40$	70
$x \le 50$	88
$x \le 60$	110
$x \le 70$	135
$x \le 80$	142
$x \le 90$	147
$x \le 100$	150

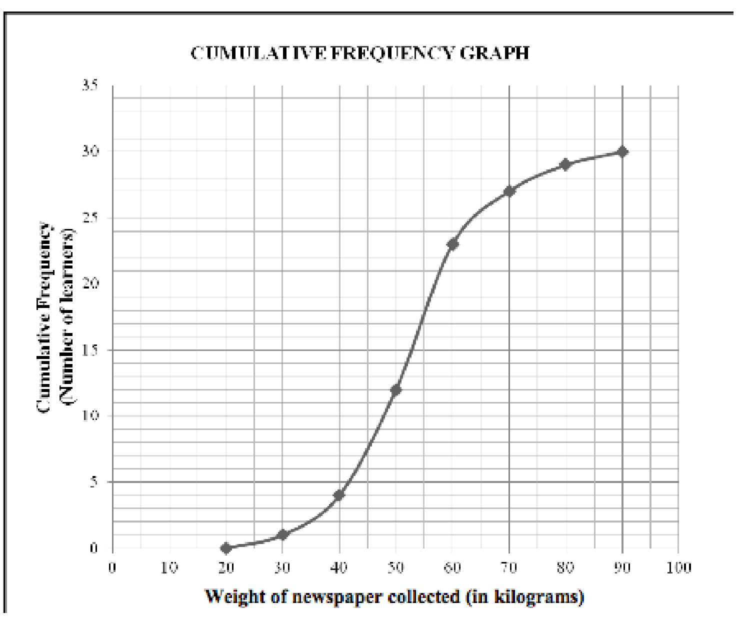
Draw the ogive (cumulative frequency graph) representing the above data. Use the ogive to approximate (1) the number of learners who scored less than 85%. (2) Approximate the interquartile range (show ALL calculations). [(3) Approximate the fortieth percentile and the median...I added these question, but making students think about other percentiles including the median, aka, the second quartile, first can help…. Think TILES!!! We can do this over and over again with a few ogives...]

comment: it probably goes without saying these are really important analytic type questions that demand students realize the significance of the numbers they are calculating...

S4 november 2012 matric

QUESTION 4

As part of an environmental awareness initiative, learners of Greenside High School were requested to collect newspapers for recycling. The cumulative frequency graph (ogive) below shows the total weight of the newspapers (in kilograms) collected over a period of 6 months by 30 learners.



Determine the modal class of the weight of the newspapers collected. Determine the median weight of the newspapers collected by this group of learners. How many learners collected more than 60 kg of newspaper? note: encourage to backtrack and create frequency distribution? emphasize count v. score?

THE SET OF REAL NUMBERS

- Real Numbers
  - Rational Numbers
    - Integers
      - Whole Numbers
        - Natural Numbers
    - Non-Integers
      - Common Fractions
      - Decimal Fractions
  - Irrational Numbers

All real numbers are either positive or negative, except zero which is neither.

All numbers in the set of integers, {…-3, -2, -1, 0, 1, 2, 3...} are either odd or even. Zero is an even number.

The set of whole numbers can be written {0, 1, 2, 3...}

The set of natural numbers, {1, 2, 3, 4...} is the set of positive integers and is also referred to as the set of counting numbers.

The set of irrational numbers includes numbers such as $\sqrt{2}$, $\sqrt[3]{17}$, π, e and any non-terminating, non-repeating decimal.

The set of common fractions includes numbers such as 3/7, -14/3, $3\frac{2}{5}$ etc. A fraction whose numerator is larger than its denominator is sometimes referred to as an "improper" fraction (a poor choice of words!).

The set of decimal fractions (usually simply referred to as decimals) contains any terminating decimal (i.e. 3.75, 0.0027) or repeating decimal (i.e. 4.767676..., $3.\overline{5}$).

Write each fraction as a decimal.

1. $\frac{3}{5}$ ________ 2. $\frac{7}{8}$ ________ 3. $\frac{7}{9}$ ________

4. $\frac{5}{16}$ ________ 5. $\frac{1}{6}$ ________ 6. $\frac{5}{8}$ ________

7. $\frac{1}{3}$ ________ 8. $\frac{2}{3}$ ________ 9. $\frac{9}{10}$ ________

10. $\frac{7}{11}$ ________ 11. $\frac{9}{20}$ ________ 12. $\frac{3}{4}$ ________

13. $\frac{4}{9}$ ________ 14. $\frac{9}{11}$ ________ 15. $\frac{11}{20}$ ________

Write each decimal as a mixed number or fraction in simplest form.

16. 0.6 ________ 17. 0.45 ________ 18. 0.62 ________

19. 0.8 ________ 20. 0.325 ________ 21. 0.725 ________

22. 4.75 ________ 23. 0.33 ________ 24. 0.925 ________

25. 3.8 ________ 26. 4.7 ________ 27. 0.05 ________

28. 0.65 ________ 29. 0.855 ________ 30. 0.104 ________

31. 0.47 ________ 32. 0.894 ________ 33. 0.276 ________

Order from least to greatest.

34. $0.\overline{2}, \frac{1}{5}, 0.02$

____

35. $1.\overline{1}, 1\frac{1}{10}, 1.101$

____

36. $\frac{6}{5}, 1\frac{5}{6}, 1.\overline{3}$

____

37. $4.\overline{3}, \frac{9}{2}, 4\frac{3}{7}$

____

38. A group of gymnasts were asked to name their favorite piece of equipment. 0.33 of the gymnasts chose the vault, $\frac{4}{9}$ chose the beam, and $\frac{1}{7}$ chose the uneven parallel bars. List their choices in order of preference from greatest to least.

____

WEEK 1

Use a number line to find each sum.

1. $8 + (-4)$ ______
2. $2 + (-3)$ ______
3. $7 - 6$ ______
4. $(-4) + (-8)$ ______
5. $3 + (-2)$ ______
6. $15 + (-8)$ ______

Find each sum.

7. $-2 + (-3)$ ______
8. $8 - 7 + 4$ ______
9. $8 + (-5)$ ______
10. $15 + (-3)$ ______
11. $-16 + 8$ ______
12. $7 + (-10)$ ______
13. $-9 + (-5)$ ______
14. $-12 + 14$ ______

Find each difference.

15. $9 - 26$ ______
16. $-4 - 15$ ______
17. $21 - (-7)$ ______
18. $27 - (-16)$ ______
19. $-16 - (-43)$ ______
20. $47 - 19$ ______
21. $-156 - 98$ ______
22. $-192 - 47$ ______
23. $0 - (-51)$ ______
24. $-63 - 89$ ______
25. $-12 - (-21)$ ______
26. $92 - (-16)$ ______

Use >, <, or = to complete each statement.

27. $-9 - (-11)$ ___ 0
28. $-17 + 20$ ___ 0
29. $11 - (-4)$ ___ 0
30. $28 - 19$ ___ 0
31. $52 + (-65)$ ___ 0
32. $-28 - (-28)$ ___ 0

Solve.

33. The highest and lowest temperatures ever recorded in Africa are 136°F and −11°F. The highest temperature was recorded in Libya, and the lowest temperature was recorded in Morocco. What is the difference in these temperature extremes?

______

34. The highest and lowest temperatures ever recorded in South America are 120°F and −27°F. Both the highest and lowest temperatures were recorded in Argentina. What is the difference in these temperature extremes?

______

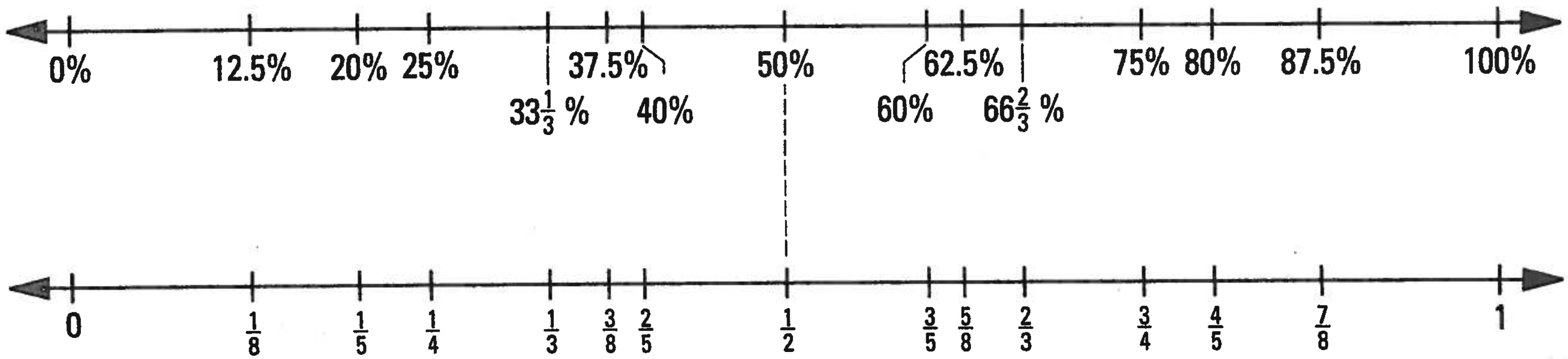
WEEK 1

FRACTION/DECIMAL/PERCENT CHART

Fraction	Decimal	Percent
1/8		
	.375	
		15%
3/5		
	.04	
		62.5%
3/2		
	$.\overline{6}$	
		40%
4/25		
	1.25	
		$22\ ^{2}/_{9}$ %

WEEK 1

Percents on a Number Line



5

Complete the chart.

Percent	12.5%		25%			40%	50%	60%		$66\frac{2}{3}$%			87.5%
Fraction		$\frac{1}{5}$		$\frac{1}{3}$	$\frac{3}{8}$		$\frac{1}{2}$		$\frac{5}{8}$		$\frac{3}{4}$	$\frac{4}{5}$	
Decimal							.5						

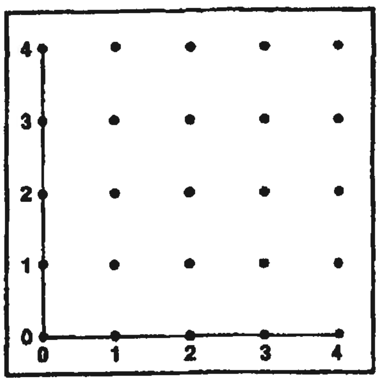
WEEK 1

Classifying Triangles by Their Angles

Triangles are also sometimes classified according to the measurement of their angles.

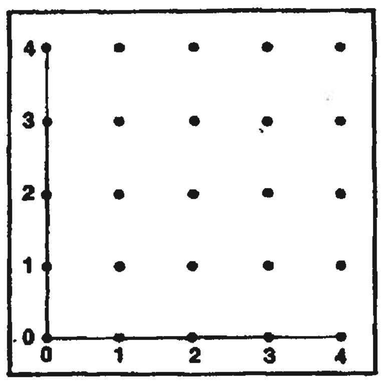
Acute triangle:	all 3 angles have a measure less than 90°.
Obtuse triangle:	one angle measures more than 90°.
Right triangle:	one angle measures 90°.

Plot the ordered pairs for the vertices (corners) of each triangle below. Classify each triangle by circling the category to which it belongs. Use a protractor to help when necessary.



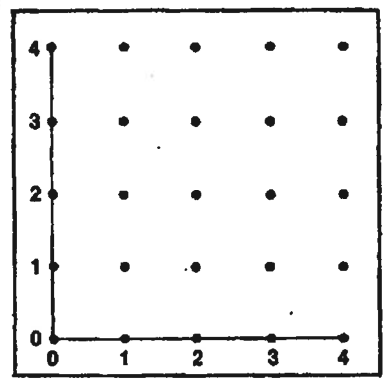
1. (0,3) (2,4) (4,2)

Acute
Obtuse
Right



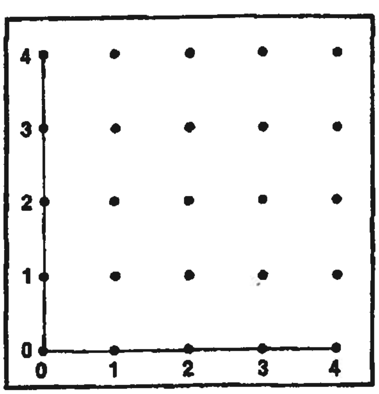
2. (1,1) (3,1) (1,4)

Acute
Obtuse
Right



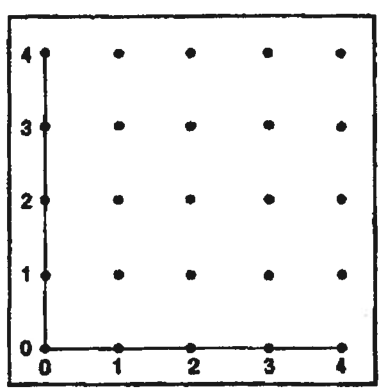
3. (1,2) (3,2) (2,4)

Acute
Obtuse
Right



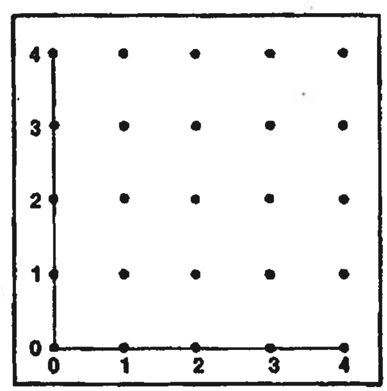
4. (2,4) (4,4) (4,1)

Acute
Obtuse
Right



5. (1,1) (2,2) (1,3)

Acute
Obtuse
Right



6. (2,2) (4,4) (0,3)

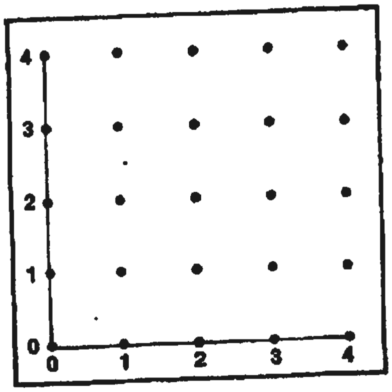
Acute
Obtuse
Right

6

WEEK 1

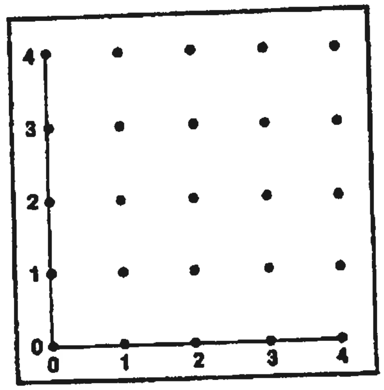
Classifying Quadrilaterals

Plot the ordered pairs for the vertices (corners) of each quadrilateral below. Classify each figure by circling <u>all</u> of the categories to which it belongs. Use a metric ruler and protractor to help when necessary.



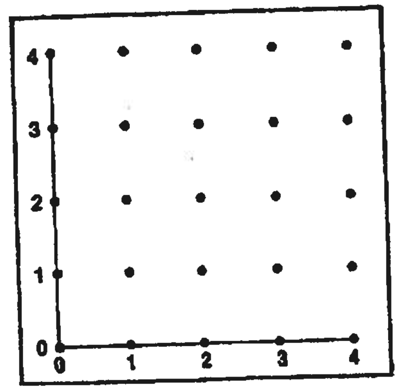
1. (1,1) (1,3) (3,3) (3,1)

Square Rectangle Trapezoid Parallelogram Quadrilateral



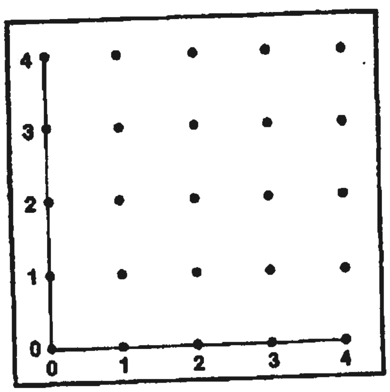
2. (1,0) (2,1) (3,0) (4,1)

Square Rectangle Trapezoid Parallelogram Quadrilateral



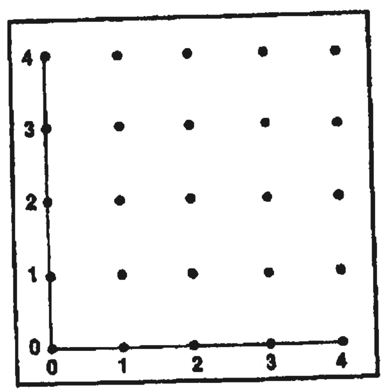
3. (0,1) (1,3) (4,1) (3,3)

Square Rectangle Trapezoid Parallelogram Quadrilateral



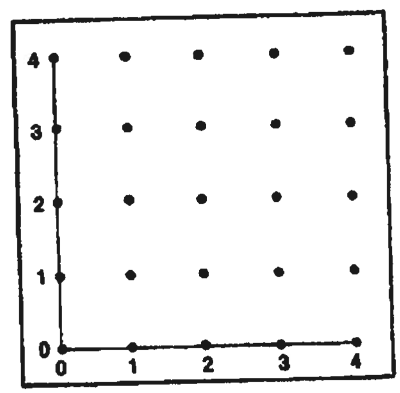
4. (0,4) (1,0) (3,0) (4,4)

Square Rectangle Trapezoid Parallelogram Quadrilateral



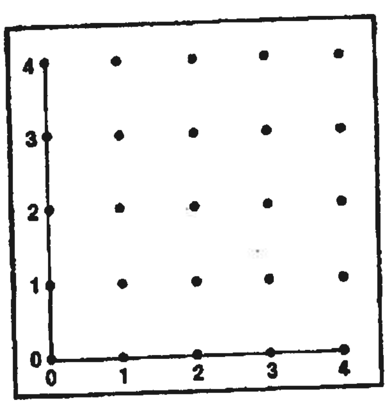
5. (1,0) (4,0) (4,3) (1,3)

Square Rectangle Trapezoid Parallelogram Quadrilateral



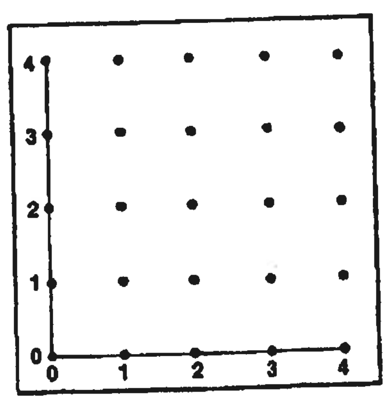
6. (3,3) (4,4) (3,2) (4,1)

Square Rectangle Trapezoid Parallelogram Quadrilateral



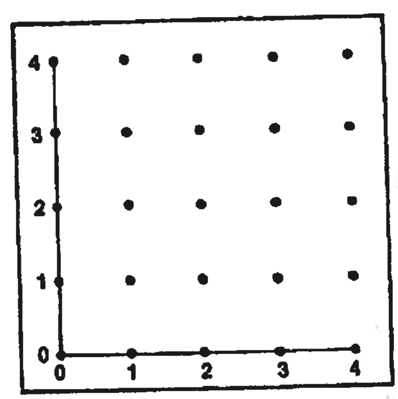
7. (0,2) (2,0) (2,4) (4,2)

Square Rectangle Trapezoid Parallelogram Quadrilateral



8. (0,0) (0,4) (3,4) (3,0)

Square Rectangle Trapezoid Parallelogram Quadrilateral

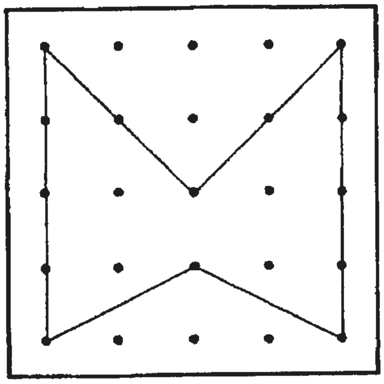


9. (2,0) (3,0) (2,4) (3,4)

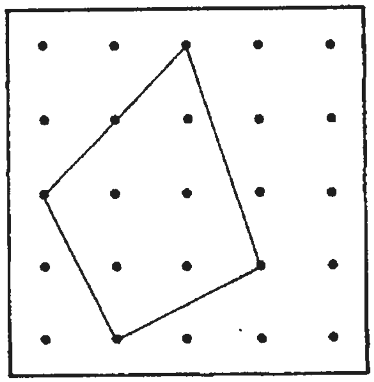
Square Rectangle Trapezoid Parallelogram Quadrilateral

Area of Polygons

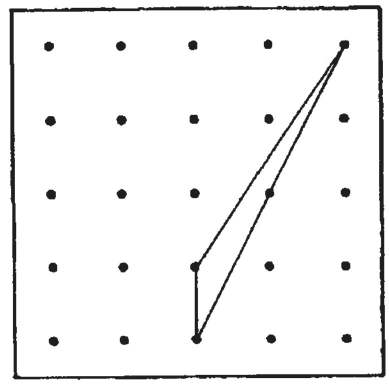
Use [illegible] strategy to find the area of each polygon. Show the rectangle you use in each case to surround the original figure.



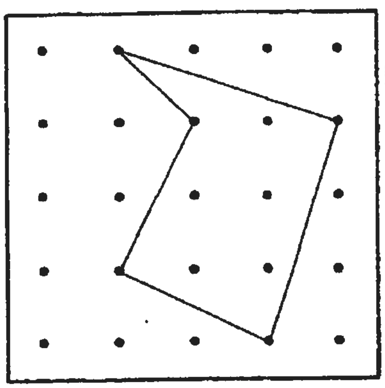
1. Area = ________________



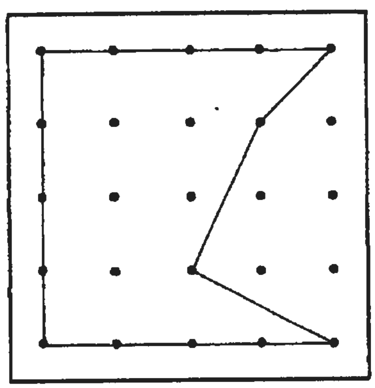
2. Area = ________________



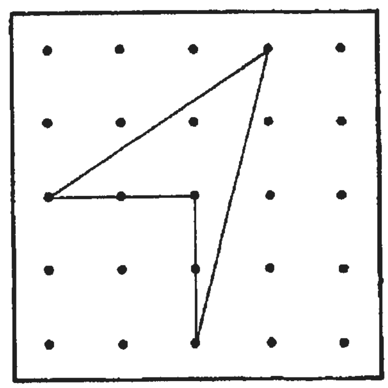
3. Area = ________________



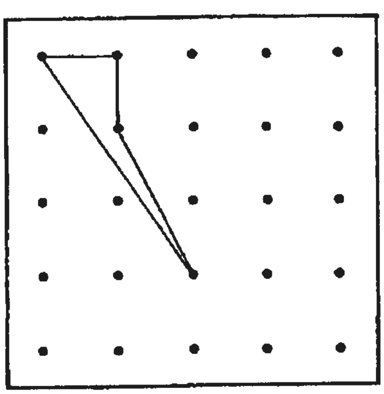
4. Area = ________________



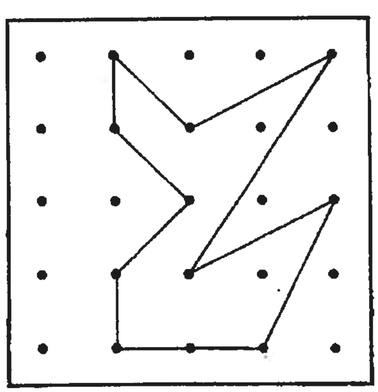
5. Area = ________________



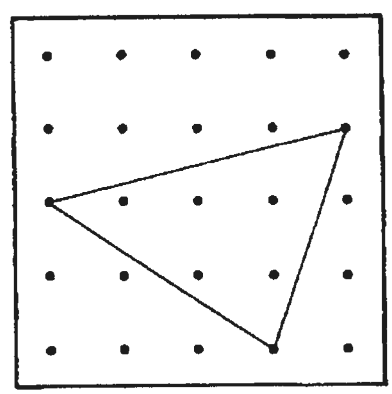
6. Area = ________________



7. Area = ________________



8. Area = ________________



9. Area = ________________

WEEK 1

✔ Classifying Quadrilaterals Check List ✔

Place a check in each box if the property applies to the quadrilateral named.

	Quadrilateral	Trapezoid	Isosceles trapezoid	Kite	Parallelogram	Rectangle	Rhombus	Square
4 sides								
only 2 ≅ sides								
2 pairs ≅ sides								
4 ≅ sides								
only 2 ∥ sides								
2 pairs ∥ sides								
1 diagonal is bisected								
both diagonals are bisected								
diagonals ≅								
diagonals ⊥								
opposite ∠s ≅								
consecutive ∠s are supplementary								
diagonals bisect only 1 pair of ∠s								
diagonals bisect 2 pairs of ∠s								
1 diagonal is a ⊥ bisector								
both diagonals are ⊥ bisectors								
4 ≅ right Δs								
4 ≅ isosceles right Δs								

Tests for Divisibility

(To determine whether a number is a factor of a given number)

A number is divisible by	If
2	the last digit of the given number is an even number.
3	the sum of the digits of the given number is itself divisible by 3.
4	the number determined by the last two digits of the given number is itself divisible by 4.
5	the last digit of the given number is 0 or 5.
6	the given number is divisible by both 2 and 3 (use the above tests).
8	the number determined by the last three digits of the given number is itself divisible by 8.
9	the sum of the digits of the given number is itself divisible by 9.
10	the last digit of the given number is 0.
12	the given number is divisible by both 3 and 4 (use the above tests).

Week 1

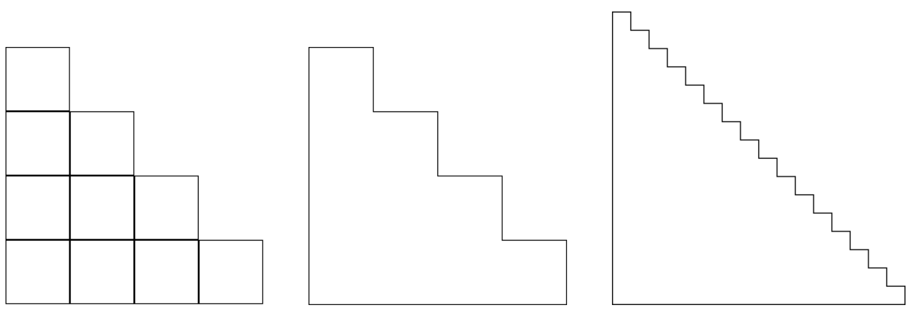
Grade 10-12: Problem Solving

Problem Solving Strategies

1. Act it out
2. Draw a diagram
3. Make a table
4. Make a graph
5. Work backwards
6. Systematize the counting process
7. Look for a pattern
8. Find a rule
9. Make the problem simpler

Staircase problem

Find the area:



A. B. C. n x n

Checkerboard

How many squares are there on an 8x8 checkerboard? Squares must follow the gridlines of the checkerboard. They can be any size and can overlap.

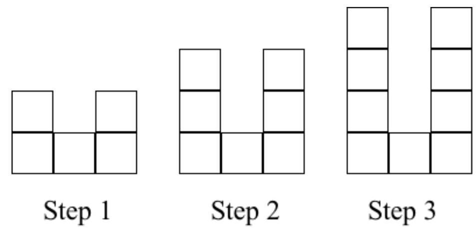
Develop a formula or rule for finding the number of squares on checkerboards of any dimension, (such as a 37x37 checkerboard)

Tower of Hanoi

Do you know the game called the Tower of Hanoi? Model the game. What is the least number of moves for a 3-tower game? A 4-tower game? Can you generalize?

Week 1

"U"



Draw step 4.

Predict what the 10^{th} step of the pattern would look like. What is a rule or formula that can help us find the n^{th} step of the pattern?

Pick a number

Pick a number. Double the number. Add 9. Add your original number. Divide by 3. Add 4. Subtract your original number. Did you get 7?

Locker Problem

A school has 1500 lockers numbered sequentially from 1 to 1500. As the students entered school one morning they did the following:

The first student opened every locker.

The second student closed every even-numbered locker.

The third student changed every locker that was a multiple of 3. That is, if the locker was open she closed it and if the locker was closed she opened it.

The fourth student changed every locker that was a multiple of 4, the fifth student changed every locker that was a multiple of 5, and so forth, until 1500 students had entered the building.

After all 1500 students had entered, which doors were open and which were closed?

Painted Cubes

1. The outside of a 3x3x3 cube is painted and then cut into 27 unit cubes. How many unit cubes have paint on only:

a.) 3 faces

b.) 2 faces

c.) 1 face

d.) 0 faces

2. Consider painting a 4x4x4 cube and then cutting it into 64 unit cubes. Answer questions a-d above.

3. Make a table and determine the answers to a-d above for a painted and cut-up cube *n*x*n*x*n*.

volume

Take two 8 by 10 sheets of paper. Using the first sheet join the two shorter sides with tape and form the shape into a short wide cylinder. Take the second sheet and join with tape the two longer sides to form a tall narrow cylinder. Which cylinder will have the greater volume or will the volumes be the same? [From the *Arithmetic Teacher,* February, 1992.]

Week 1

percent

A group of 200 learners took a test and 98% were successful. How many more successful learners would need to be added to this group in order to have a 99% success rate?

wow!

Write down 6 random single digit integers in row 1. Pick any integer from 1 to 9 and write that digit 6 times to make row 2. Add row 1 and row 2 to make row 3, but write only the units digit if the sum is more than 9. Add row 2 and row 3 to make row 4. Continue adding the last 2 rows until you have row 17. Explain why this works.

Order of Operations Simplified

The issues:

Problem 1.) Simplify $1 \div 2 \times 3$.

Problem 2.) Simplify $8 - 4 + 2$.

Problem 3.) Simplify $12 \times 4 + 2$.

New Procedure:
Replace all division with multiplication and all subtraction with addition.
Then do all multiplication, then all addition.

1.) Simplify $10 \times 12 \div 4$.

2.) Simplify $16 + 8 \div 4 - 2$.

3.) Simplify $4 \times 4 \div 8 + 9 + 1 \div 2 \times 3 - 4 + 6$.

4.) Simplify $7 - 1 \times 0 + 3 \times \frac{1}{3}$.

5.) Simplify $6 \div 2(1+2) = 6 \times \frac{1}{2} \times (1+2) = 6 \times \frac{1}{2} \times 3 = 9$.

6.) Simplify $10 \times 12 \div 4 - 3 + 6 \div 2 \times 5 + 30 - 7 \times 8 \div 4$.

7.) Simplify $\sqrt{(5-2)^2 + (3+7)^2}$.

8.) Insert parentheses to make the following a true statement: $5 - 2 \times 1 + 4 \div 6 = 5$.

painting

a.) Thokozani can paint a room in 6 hours. How much of the room can he paint in one hour?

b.) Toto can paint a room in 3 hours. How much of the room can she paint in one hour?

c.) When Thokozani and Toto work together, how much of the room can they paint in one hour?

d.) In how many hours can Thokozani and Toto paint one room?

fractions

a.) Write 2 factions whose difference is 2/13, such that when each fraction is in lowest terms the denominators are different.

b.) A man died with 17 horses, but his will said the eldest child gets ½ of his property, the middle child gets 1/3 and the youngest 1/9. The lawyer rode in on his horse and said, "I'll loan you mine, then we have 18. The eldest got 9, the middle one got 6 and the youngest got 2, for a total of 17, and the lawyer took back his horse. What happened? Find other numbers for which this works.

Week 1

Grade 7-9: Introduction to Algebra

Order of Operations Simplified

The issues:

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6.) Simplify $10 \times 12 \div 4 - 3 + 6 \div 2 \times 5 + 30 \times 7 \times 8 \div 4$.

7.) Simplify $\sqrt{(5-2)^2 + (3+7)^2}$.

8.) Insert parentheses to make the following a true statement: 5 - 2 x 1 + 4 $\div$ 6 = 5.

Pick a number

a.) Pick a number. Add 5. Double the result. Subtract 4. Divide by 2. Subtract the number you started with. Prove that the answer is always 3.

b.) Pick a number. Double the number. Add 9. Add your original number.
Divide by 3.Add 4. Subtract your original number. Prove that you get 7.
Invent other problems like this using your skills in algebra.

phone

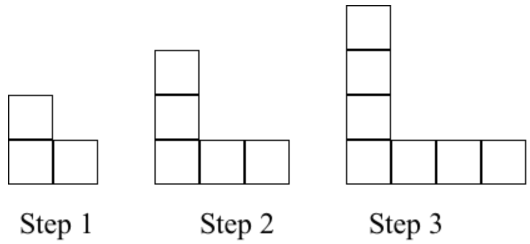
Ignore your area code & using only 7 digit phone:

1) key-in the first 3 digits of your phone number into the calculator
2) multiply by 80
3) then plus 1
4) multiply by 250
5) plus last four digits of phone number
6) plus last four digits of phone number again
7) minus 250
8) divide by 2 at last

Is the answer your phone number?

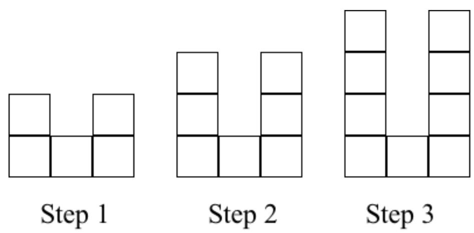
Week 1

“L”



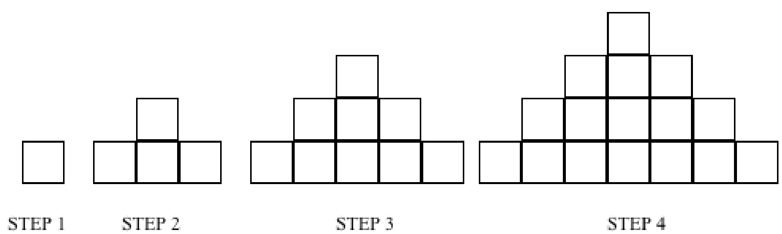
Draw step 4.
Predict what the 10th step of the pattern would look like. What is a rule or formula that can help us find the nth step of the pattern?

“U”



Draw step 4.
Predict what the 10th step of the pattern would look like. What is a rule or formula that can help us find the nth step of the pattern?

Double stairs



Draw step 5.
Predict what the 10th step of the pattern would look like. What is a rule or formula that can help us find the nth step of the pattern?

students

A university has 6 times as many students as professors. If S is the number of students and P is the number of professors, write an equation expressing the relationship between S and P.

perfume

A bottle of perfume costs 70 rand. The perfume in the bottle is 10 times the value of the bottle. How much is the bottle worth?

animals

a.) The ratio of birds to dogs is 3:2. The ratio of cats to birds is 4:7. There are 87 animals. How many of each?

b.) A pen has goats and ducks. There are 44 feet and 30 eyes. How many goats are there?

Week 1

learners

A group of 200 learners took a test and 98% were successful. How many more successful learners would need to be added to this group in order to have a 99% success rate?

fractions

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Linear & Quadratic Equations

1. $4(-3+x)+5=-10(x-4)-14x$

2. $\frac{1}{2}x-\frac{5}{3}=-\frac{1}{2}x+\frac{19}{4}$ (keep work in fraction form)

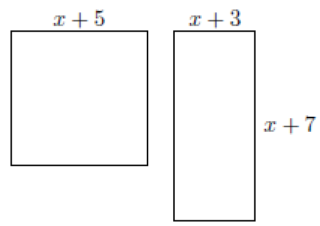
3. A rectangular plot of land has a width that is three times its length. If the perimeter is 95m, how long is each side?

4. When a third of a number is subtracted from half of the same number, 60 is the result. Set up a linear equation that describes the situation and find the number.

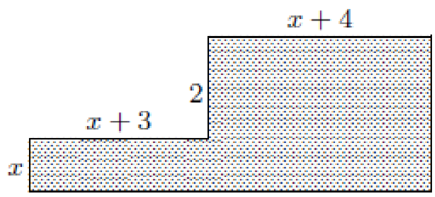
5. The sum of four consecutive integers is 2174. What are the integers?

6. There are 396 persons in a theater. If the ratio of women to men is 2:3, and the ratio of men to children is 1:2, how many men are in the theater?

7. For what values of x will the square and the rectangle shown at right have the same perimeter?



8. In the diagram, the dimensions of a piece of carpeting have been marked in terms of *x*. All lines meet at right angles. Express the area and the perimeter of the carpeting in terms of *x*.



9. A group of friends are planning a trip. They have 40 people signed up to go, and the resort is charging $120 for each person.

(a) Calculate how much money (revenue) the resort expects to take in.
(b) The resort manager offers to reduce the group rate of $120 per person by $2 for each additional registrant, up to a maximum number *m*. For example, if five more people were to sign up, all 45 would pay $110 each, producing revenue $4950 for the resort. Fill in the rest of the table at right, and you will discover the manager's value of *m*.
(c) Let *x* be the number of new registrants. In terms of *x*, write expressions for the total number of persons going, the cost to each, and the resulting revenue for the resort.
(d) For the resort to take in at least $4900, how many people must go on trip?

extras	*persons*	*cost/person*	*revenue*
0			
1			
2			
3			
4			
5	45	110	4950
6			
7			
8			
9			
10			
11			
12			

Quadratic equations

1. Factor the following *perfect-square trinomials*:

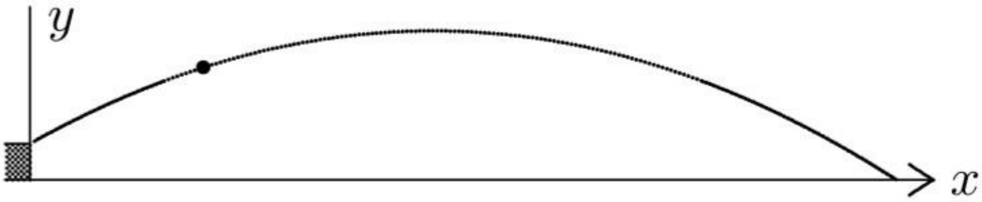
(a) $x^2 - 12x + 36$ (b) $x^2 + 14x + 49$ (c) $x^2 - 20x + 100$

As suggested, these should all look like either $(x - r)^2$ or $(x + r)^2$. State the important connection between the *coefficients* of the given trinomials and the values your found for r.

2. (Continuation) In the following, choose k to create a perfect-square trinomial:

(a) $x^2 - 16x + k$ (b) $x^2 + 10x + k$ (c) $x^2 - 5x + k$

3. Using a driver on the 8th tee, which is on a plateau 10 yards above the level fairway, Dale hits a fine shot. Explain why the quadratic function $y = 10 + 0.5x - 0.002x^2$ describes this parabolic trajectory, shown in the figure below. Why should you expect this tee shot to go more than 250 yards? Estimate the length of this shot, then use technology to find a more accurate value.



4. (Continuation) To find the length of this shot without technology, you must set $y = 0$ and solve for x. Explain why, and show how to arrive at $x^2 - 250x = 5000$.

(a) The next step in the solution process is to add 125^2 to both sides of this equation. Why was this number chosen?

(b) Complete the solution by showing that the length of the shot is $125\sqrt{20625}$. How does this number, which is about 268.6 compare with your previous calculation?

(c) Comment on the presence of the number 125 in the answer. What is its significance?

5. Solving a quadratic equation by rewriting the left side as a perfect-square trinomial is called solving by *completing the square*. Use this method to solve each of the following equations. Leave your answers in exact form.

(a) $x^2 - 8x = 3$ (b) $x^2 + 10x = 11$ (c) $x^2 - 5x - 2 = 0$ (d) $x^2 + 1.2x = 0.28$

6. In solving an equation such as $3x^2 - 11x = 4$ by completing the square, it is customary to first divide each term by 3 so that the coefficient of x^2 is 1. This transforms the equation into $x^2 - \frac{11}{3}x = \frac{4}{3}$. Now continue to solve by completing the square method, remembering to take half of $\frac{11}{3}$, square it and add it to *both* sides of the equation. Finish the solution.

7. *Completing the square*. Confirm that the equation $ax^2 + bx + c = 0$ can be converted into the form $x^2 - \frac{b}{a}x = \frac{c}{a}$. Describe the steps. To achieve the goal suggested by the title, what should now be added to both sides of this equation?

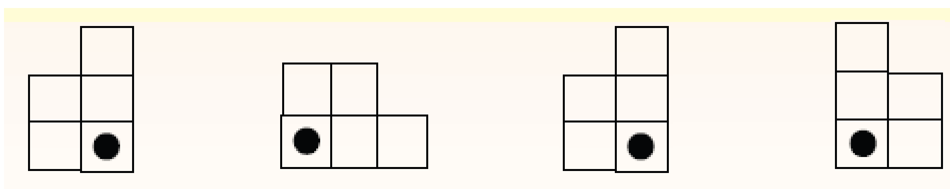
8. (Continuation) The left side of the equation $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$ can be factored as *a* perfect square trinomial. Show how. The right side of the equation can be combined over a common denominator. Show how. Finish the solution of the general quadratic equation by taking the square root of both sides of your most recent equation. The answer is called *the quadratic formula*. Apply your formula:
Solve $x^2 + 2x - 3 = 0$ by letting $a = 1$, $b = 2$, and $c = -3$.

9. Solve each of the following quadratic equations:
(a) $3x^2 - 6x = 1$ (b) $2x^2 + 8x - 18 = 0$ (c) $-10 = 0.5x - 0.002x^2$

Transformation Geometry

1. A *transformation* is a way to manipulate a point, line, or figure. The original object is called the *preimage* and the resulting one is known as the *image*. Using your own words, describe different types of transformations.

2. Explain how the first figure below could be *transformed* into the following images.

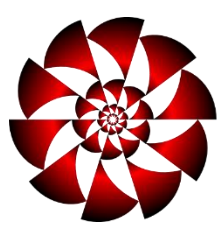


3. (Continuation) A *rotation* is a transformation that moves points so that they stay the same distance from a fixed point, known as the *centre of rotation*. Which of the figures in question 2 is a rotation?

4. (Continuation) A *translation* is a transformation that moves an object to a new position without changing its shape, size, or orientation. Which of the figures in question 2 is a translation?

5. (Continuation) A *reflection* is a transformation that has the same effect as a mirror. The line over which a figure is reflected is known as the *line of reflection*. Which of the figures in question 2 is a reflection?

6. A figure has *rotational symmetry* if an outline of the turning figure matches its original shape. In other words, if the figure looks the same after a rotation, then the figure has rotational symmetry. Explain why the following figure has rotational symmetry.



7. (Continuation) How many times can the figure above be rotated so that it looks the same before it returns to the original orientation? This is the *order of symmetry*.

8. A figure has *reflective symmetry* when one half is a mirror image. The *line of symmetry* is the line over which half of the figure is reflected. Does the figure above have reflective symmetry?

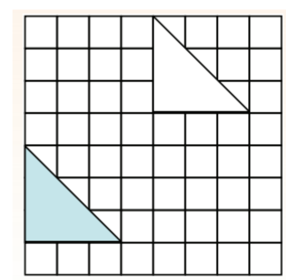
9. Draw the lines of symmetry into the figures below.

10. Explain why the word AVIVA has reflective symmetry. Can you think of other words that have reflective symmetry? What letters are not allowed in your words?

11. Explain why the figure to the right has both reflective and rotational symmetry. What is the order of rotational symmetry?

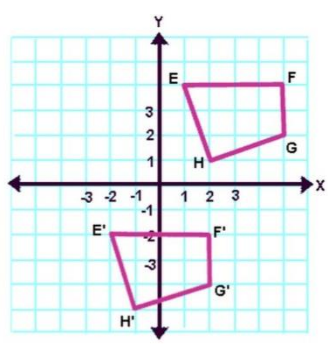
12. (Continuation) How many lines of symmetry does the figure have?

13. How has the shaded shape below been transformed? Be as specific as possible in your answer.



14. (Continuation) A triangle has vertices at (0,1), (0,4) and (1,3) and is translated so that the vertex which used to be at (0,1) is now at (4,5). Draw a picture of this translation. How could you describe this translation?

15. How has EFGH been transformed?

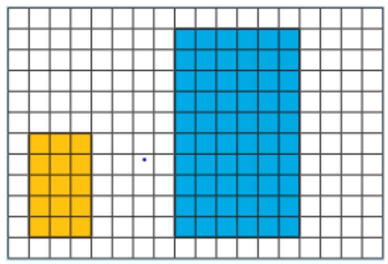


16. Suppose that the smaller rectangle below has been *enlarged* to form the larger one. (a) What are the dimensions of the smaller rectangle?

    (b) What are the dimensions of the larger rectangle? (c) Fill in the blanks:

    i. The length of the larger rectangle is ________________the length of the smaller one. ii. The width of the larger rectangle is __________________the width of the smaller one.

    (d) By how much has the smaller rectangle been enlarged to form the larger one?



17. (Continuation) Now suppose that the larger rectangle has been *reduced* to form the smaller one. Fill in the blanks.

    (a) The length of smaller rectangle is ________________the length of the larger one.

    (b) The width of the smaller rectangle is ________________the width of the larger one. (c) The larger rectangle has been reduced by _____________________to form the smaller one.

18. (Continuation) What is the area of the smaller rectangle? What is the area of the larger one? How do those areas compare?

<u>Geometric Transformation Challenge Problems</u>

1. If you reflect a figure over a line and then reflect that image over a parallel line, what is the resulting transformation?

2. (Continuation) If you reflect a figure over a line and then reflect the image over another line which intersected the first, what is the resulting transformation?

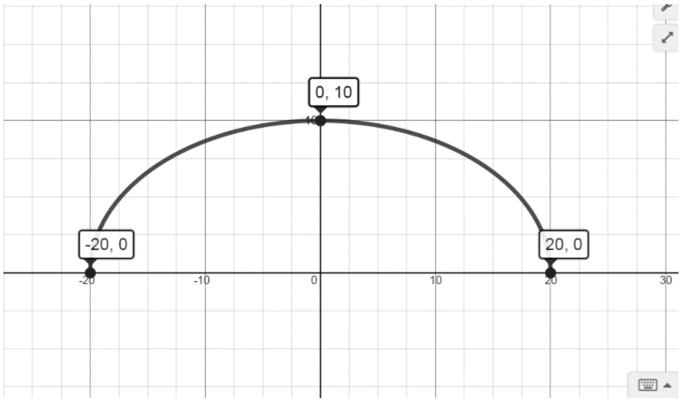
3. The image of an enlargement of a square by three is a larger square. By what percentage has the perimeter increased? By what percentage has the area increased?

4. On a road map of Uganda, the scale is 1 : 1 500 000. The distance on the map from Kampala to Ft. Portal is 17cm. What is the real world distance in km between these two cities?

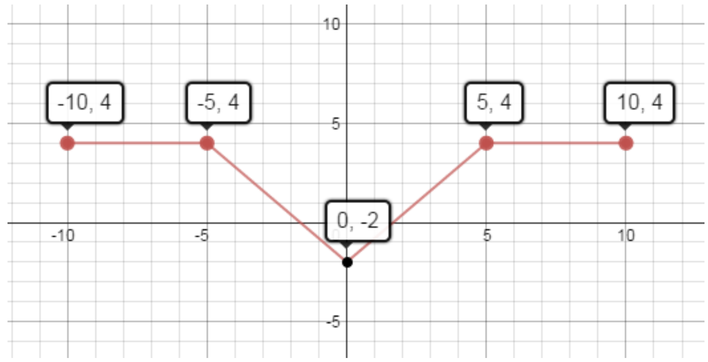
5. On a map of South Asia, Nepal looks approximately like a rectangle measuring 8.3 cm by 2.0 cm. The map scale is listed as 1 : 9 485 000. What is the approximate real world area of Nepal in km2?

<u>Functions</u>

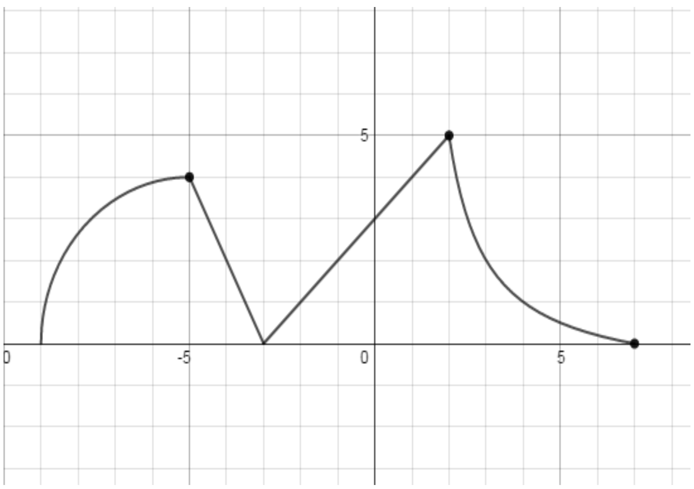
Function A



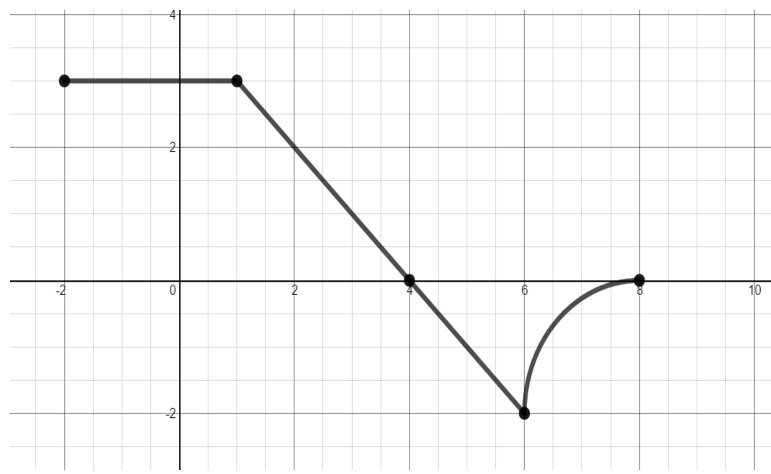
Function B



Function C



Function D



On graph paper, show the following transformations of each function

1) $g(x) = f(x) + 2$
2) $g(x) = 4f(x)$
3) $h(x) = \frac{1}{3}f(x)$
4) $g(x) = f(x-1)$
5) $g(x) = f(5x)$
6) $h(x) = f\left(\frac{1}{2}x\right)$
7) $g(x) = f(-x)$
8) $g(x) = -f(x)$
9) $2f(x) + 1$
10) $-f(-x)$

When you have multiple transformations at once, the order of transformations are:

1. Horizontal translation
2. Vertical & horizontal stretches/compressions
3. Reflections
4. Vertical translation

Using the given parent functions, describe the transformation that takes place and graph

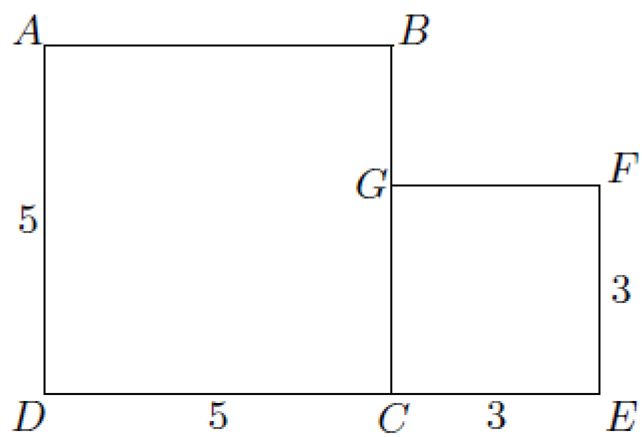
$f(x) = x$
$f(x) = |x|$
$f(x) = x^2$
$f(x) = \sqrt{x}$
$f(x) = \frac{1}{x}$
$f(x) = b^x$
$f(x) = \log x$
$f(x) = \sin x$

$h(x) = f(x) - 7$
$m(x) = 2.5f(x)$
$k(x) = 0.1f(x)$
$p(x) = f(x+5)$
$w(x) = f(2x)$
$c(x) = f\left(\frac{1}{3}x\right)$
$z(x) = f\left(\frac{1}{4}x - 10\right)$
$j(x) = -2f(2x+5) + 1$

Trigonometry

1. A 5 $\times$ 5 square and a 3 $\times$ 3 square can be cut into pieces that will fit together to form a third square.

(a) Find the length of a side of the third square.
(b) In the diagram at right, mark *P* on segment *DC* so that *PD* = 3, then draw segments *PA* and *PF*. Calculate the lengths of these segments.
(c) Segments *PA* and *PF* divide the squares into pieces. Arrange the pieces to form the third square.



2. Instead of walking along two sides of a rectangular field, Fran took a shortcut along the diagonal, thus saving distance equal to half the length of the longer side. Find the length of the long side of the field, given that the the length of the short side is 156 meters.

3. A five-foot student casts a shadow that is 40 feet long while standing 200 feet from a streetlight. How high above the ground is the lamp?

4. How far from the streetlight should the student stand, in order to cast a shadow that is exactly as long as the she is tall?

5. The diagonal of a rectangle is 15 cm, and the perimeter is 38 cm. What is the area?

6. One of the legs of a right triangle is twice as long as the other, and the perimeter of the triangle is 28. Find the lengths of all three sides, to three decimal places.

7. I am thinking of a right triangle, whose sides can be represented by $x - 5$, $2x$, and $2x + 1$. Find the lengths of the three sides.

8. Standing 50 meters from the base of a fir tree, Rory used a protractor to measure an angle of elevation of 33° to the top of the tree. How tall was the tree?

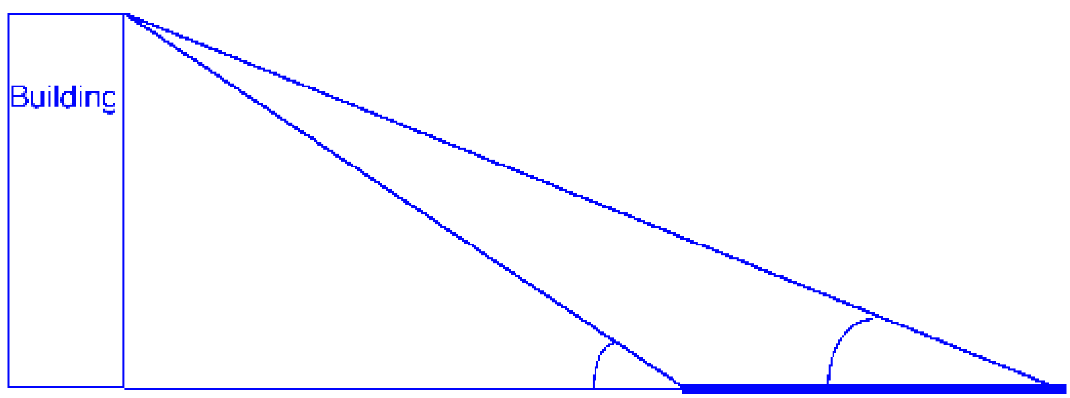
9. Out for a walk in, Morgan measures the angle of elevation to the distant tower, and gets 3.6 degrees. After walking one km directly toward the building, Morgan finds that the angle of elevation has increased to 4.2 degrees. Use this information to calculate the height of the tower, and how far Morgan is from it now

10. Standing on a cliff 380 meters above the sea, Pat sees an approaching ship and measures its angle of depression, obtaining 9 degrees. How far from shore is the ship?

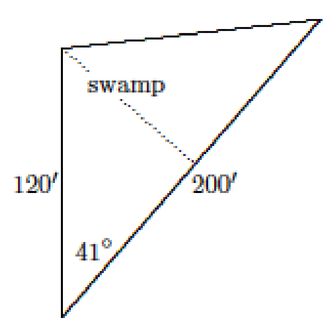
11. (Continuation) Now Pat sights a second ship beyond the first. The angle of depression of the second ship is 5 degrees. How far apart are the ships?

12.

Determine the height of a nearby building by measuring the angle of elevation from 2 different points in line with the building and measuring the distance between the 2 points.



13. A triangular plot of land has the SAS description indicated in the figure shown. Although a swamp in the middle of the plot makes it awkward to measure the altitude that is dotted in the diagram, it can at least be calculated. Show how. Then use your answer to find the area of the triangle, to the nearest square foot.

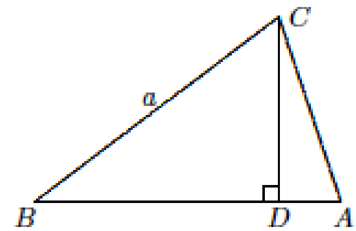


14. 3. (Continuation) Find the length of the third side of the triangle, to the nearest foot.

15. A 12.0-cm segment makes a 72.0-degree angle with a 16.0-cm segment. To the nearest tenth of a cm, find the third side of the triangle determined by this SAS information.

16. (Continuation) Find the area of the triangle, to the nearest square centimeter.

17. In the diagram shown to the right, CD is the altitude from C.
    a) Express CD in terms of angle B and side a.
    b) Express BD in terms of angle B and side a.
    c) Simplify the expression $(a \sin B)^2 + (a \cos B)^2$ and discuss its relevance to the diagram.
    d) Why was *asinB* used instead of *sinB·a*?



18. A 12.0-cm segment makes a 108.0-degree angle with a 16.0-cm segment. To the nearest tenth of a cm, find the third side of the triangle determined by this SAS information.

19. (Continuation) Find the area of the triangle, to the nearest square centimeter.

20. A segment that is *a* units long makes a C-degree angle with a segment that is *b* units long. In terms of *a, b,* and *C*, find the third side of the triangle defined by this SAS description. You have done numerical versions of this question. Start by finding the length of the altitude drawn to side *b*, as well as the length of the altitude from angle *B* onto side *b*. The resulting formula is known as the Law of Cosines.

21. (Continuation) What is the area of the triangle defined by a, b, and C?

Uncovering the sum formulas for $\mathbf{sin}(\boldsymbol{\alpha}+\boldsymbol{\beta})$ and $\mathbf{cos}(\boldsymbol{\alpha}+\boldsymbol{\beta})$

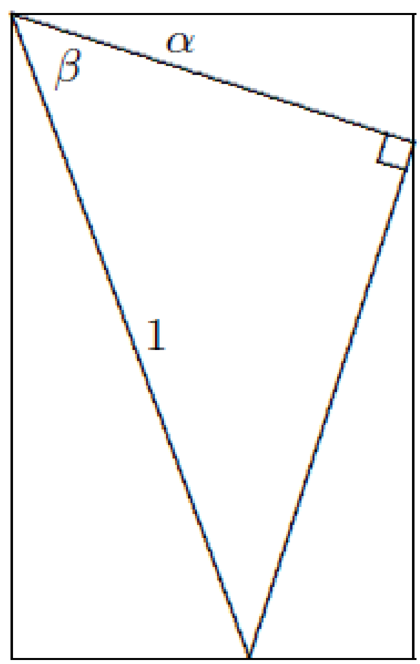
The rectangle shown has been formed by fitting together four right triangles. As marked, the sizes of two of the angles are α and β, and the length of one segment is 1.

a) Starting at the top left and going clockwise, label the vertices of the rectangle ABCD and the right triangle AEF

b) Find the two unmarked angles whose sizes are α and $\alpha + \beta$.

c) In terms of sin α, cosα, sinβ, and cosβ, determine the lengths of all segments in the diagram (start with the inside triangle).

d) Once you've labeled all the segments of the diagram, see if you can uncover the formulas for sin(α+β) and cos($\alpha + \beta$), written in terms of sin α, cosα, sinβ, and cosβ.



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