TEACHERS
ACROSS
BORDERS

MATHS

Grades 10 - 12

Bloemfontein: July 17 - 21, 2017

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EXECUTIVE FUNCTIONING (3)
OVERVIEW

Executive functioning is a set of cognitive processes a person needs in order to develop and execute a plan. They include:

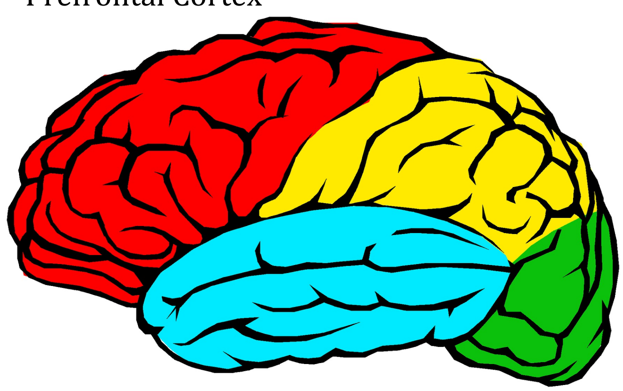
- Strategizing and planning
- Focusing attention
- Initiating appropriate behavior
- Organizing ideas and materials
- Inhibiting inappropriate responses
- Mental flexibility
- Self-monitoring emotions and actions

Signs of weak executive functioning skills:

- Disorganized
- Loses focus
- Late
- Impulsive
- Unprepared
- Unaware
- Disengaged
- Inconsistent performance/behavior
- Lack of precision/depth

The prefrontal-cortex (also knows as the pre-frontal lobe) controls executive functioning. It is the "newest" part of the human brain and is unique to humans.

Prefrontal Cortex



The pre-frontal cortex, which controls executive functioning, becomes fully developed in human beings between the ages of 21 and 29. On average, the pre-frontal cortex of women develops earlier than men.

EXECUTIVE FUNCTIONING, CTD.

Executive functioning is needed to process:

- Language
- Future events
- Abstract Concepts
- New experiences/information

Executive functioning is less burdened when dealing with situations that are:

- Tangible
- Personal
- Immediate
- Intense
- Meet a basic need
- Familiar

Factors that affect executive functioning:

- Neurological maturation
- Hormonal fluctuations
- Fear
- Emotional distress
- Concussion
- Lack of sleep
- Fatigue
- Poor nutrition

Executive functioning plays a key role in educational performance, particularly in math.

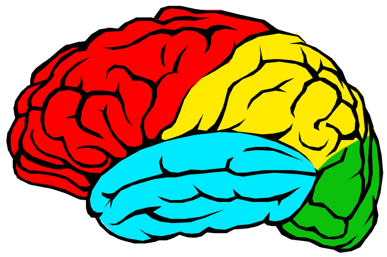
Since students' brains develop at different rates, students who are the same age will vary greatly in their ability to perform tasks that rely heavily on executive functioning skills.

The pre-frontal cortex requires a lot of energy to function properly. Tasks that rely heavily upon executive functioning can lead to students becoming mentally fatigued.

Some students have difficulty with a low supply of blood flow (fuel) to their prefrontal cortex, which affects their executive functioning.

When the body is in "fight or flight mode," it redirects blood flow away from the prefrontal cortex. Therefore, students who are afraid or suffer from emotional distress have difficulty with executive functioning.

NEUROPLASTICITY (4)
OVERVIEW



The neurological connections in the brain are constantly growing and changing.

Not only do our brains control our actions, but also our actions affect our brains.

What we do creates new neurons and neural pathways in the brain or reinforces pre-existing pathways.

The more a student actively engages in an activity, the more his/her brain will develop the ability to support that activity later.

The more regions of the brain that are involved in an activity, the more complex and long-lasting the neural connections that develop will be. Students learn best by engaging material in multiple ways:

- Write it
- Hear it
- Talk about it
- Feel it
- Act it
- Picture it
- Color it
- Connect it
- Sing it

It is better for a student to complete an activity with help, than to not engage in the activity at all. Active engagement of any kind, even with support, helps neurological connections in the brain to form, which will support the possibility of future mastery.

VYGOTSKY'S ZONE OF PROXIMAL DEVELOPMENT (ZPD) (1)
OVERVIEW

MASTERY/COMPETENCE
ZPD
STUDENT'S CURRENT LEVEL OF ABILITY

Soviet psychologist, Lev Vygostky, described learning as happening in the space between a student's current level of ability and the desired level of mastery. He called this gap between a student's current level and the level of mastery the "zone of proximal development" (ZPD).

Vygotsky noted that if the ZPD was too big, a student would feel overwhelmed, frustrated, and most likely quit. However, if the ZPD was too small, the student would lose interest because it could be traversed without much effort or growth.

Vygotsky used the term "scaffolding" to describe the types of supports teachers put in place to help students move from their current level of ability, across the ZPD, to mastery.

Vygotsky created this model while studying individual students. The challenge for classroom teachers is managing every student's unique ZPD within the class, keeping the ZPD at an optimal level of learning for a variety of students with very different base levels of ability.

Understanding how to utilize students' neurological strengths position students with an optimal ZPD.

THE ICEBERG MODEL OF TEACHING (2)

OVERVIEW

Just as 90% of an iceberg is below the surface of the water, much of what affects student learning is not visible.

Neurological diversity exists between all of us. It is caused by factors such as:

- Genetics
- Physiology
- Prenatal development
- Life experiences
- Gender
- Maturation
- Biochemistry

TEACHER REFLECTION (5)

1. What subject do you teach? Why? What is important about it?

2. What do you want your students to leave your class being able to do and know?

3. Besides the content of your course, what else do you teach your students? Why?

4. How much of your teaching time is dedicated to teaching skills, habits, and information that is not directly linked to your curriculum?

5. What are some of the obstacles to learning that your students face? How do you work to support your students?

NEURO-MAXIMIZING TEACHING PLAN WORKSHEET (6)

LESSON NAME:

OBJECT OF LESSON:

PREREQUISITE SKILLS:

EXECUTIVE FUNCTIONING BURDEN: Low 1 2 3 4 5 6 7 8 9 10 High
(abstract concept, new, detail oriented, not immediately gratifying, etc)

OTHER POSSIBLE DIFFICULTIES/OBSTACLES TO LEARNING:

STRATEGIES TO SUPPORT NEUROLOGICAL DEVELOPMENT AND LEARNING:
(choose as many as seem relevant)

SUPPORT EXECUTIVE FUNCTIONING	SUPPORT NEURAL CONNECTIONS
• Tangible • Personal • Immediate • Intense • Meets a basic need • Familiar • Safe • Fun	• Write it • Hear it • Talk about it • Feel it • Act it • Picture it • Color it • Connect it • Sing it

1.

2.

3.

4.

5.

NEURO-MAXIMIZING TEACHING PLAN WORKSHEET

LESSON NAME:

OBJECT OF LESSON:

PREREQUISITE SKILLS:

EXECUTIVE FUNCTIONING BURDEN: Low 1 2 3 4 5 6 7 8 9 10 High
(abstract concept, new, detail oriented, not immediately gratifying, etc)

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1.

2.

3.

4.

5.

Grades 10-12 Priority Topics:
Calculus and Financial Math

Differential Calculus

1. A rock is dropped off a cliff and its height above ground is given by the function $f(t) = 100 - 4.9t^2$, where t is measured in seconds.
   (a) What is the rock's height 3 seconds after it is dropped?
   (b) What is the rock's height 5 seconds after it is dropped?
   (c) What was the rock's average speed over the 2-second interval from 3 seconds to 5 seconds? Remember that *average speed* is total distance traveled divided by total time elapsed.
   (d) What was the rock's average speed during the time interval $t = 3$ and $t = 4$?
   (e) What was the rock's average speed during the time interval from $t = 3$ to $t = 3.1$?
   (f) How do you think we can find the exact speed at which the rock is falling at $t = 3$ seconds?

2. (Continuation) Let $f(x) = 100 - 4.9x^2$. Let $P = (3, p)$ be a point on the curve, and let $Q = (3 + h, q)$ be a nearby point on the curve (whose x-coordinate is $3 + h$). Carry out the following computations. You may use your calculator to confirm your answer, but you must show your algebra for credit.
   (a) Find the value of p.
   (b) Express q in terms of h.
   (c) Write an expression, in terms of h, for the slope of the line through P and Q.
   (d) What does your expression in the previous part represent in terms of the falling rock? What are the units?

3. (Continuation) In order to find the slope *at* P, we can make Q infinitely close to P, which means that h needs to go towards 0. Calculate

$$\lim_{h\to 0} \frac{q-p}{(3+h)-3} = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h}$$

   What does this limit represent in terms of the falling rock? What are the units?

4. (Continuation) Write and evaluate a limit expression to find the speed of the rock at any time x.

5. (*2016 Feb-March Paper 1*) Determine $f'(x)$ from first principles if $f(x) = -x^2 + 4$.

6. Use first principles to find $f'(x)$ if $f(x) = -3x^3$.

7. Use first principles to find $f'(x)$ if $f(x) = \frac{1}{x}$.

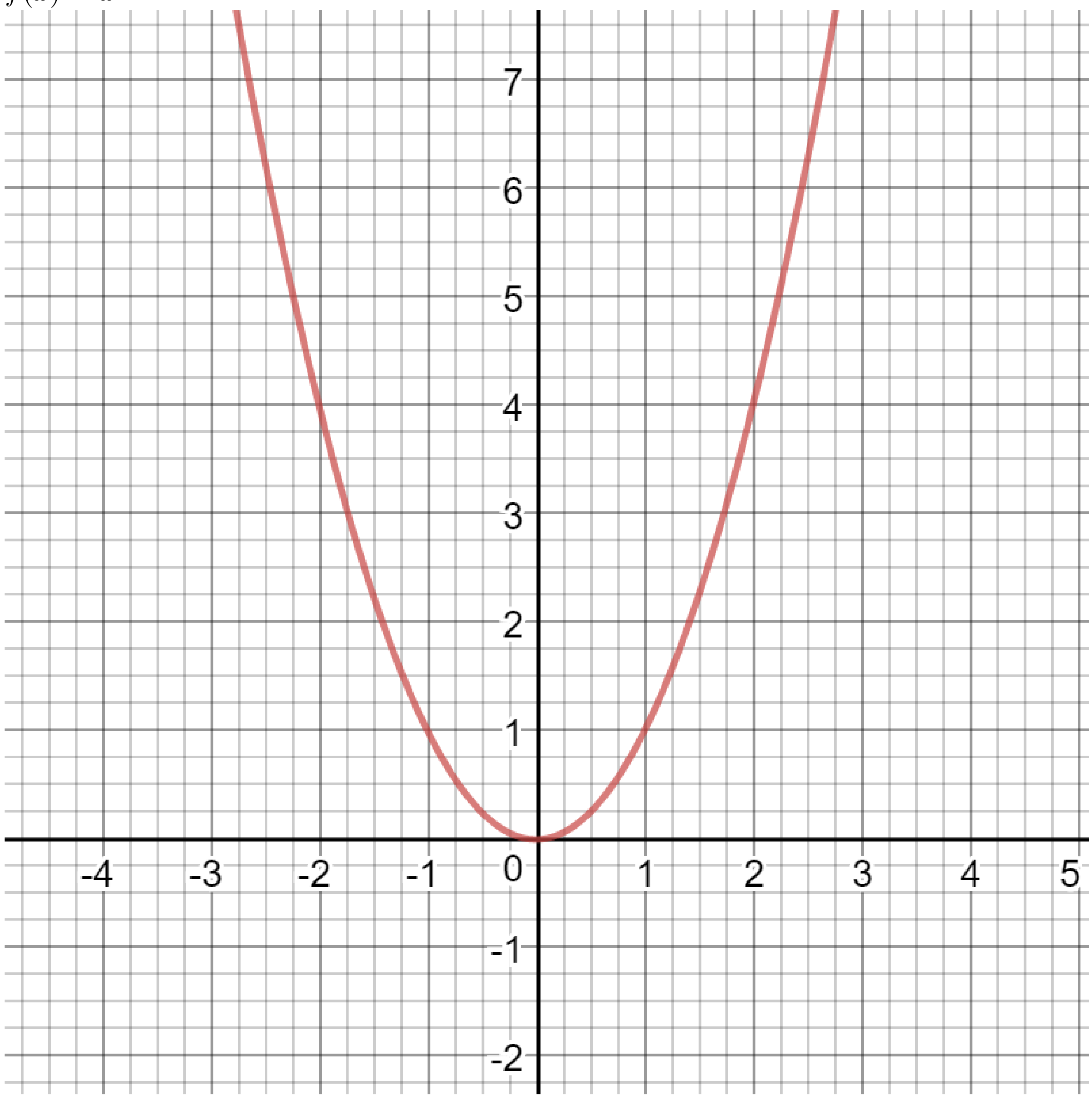
8. (*2016 Feb-March Paper 1*) Determine the derivative of:

   (a) $y = 3x^2 + 10x$

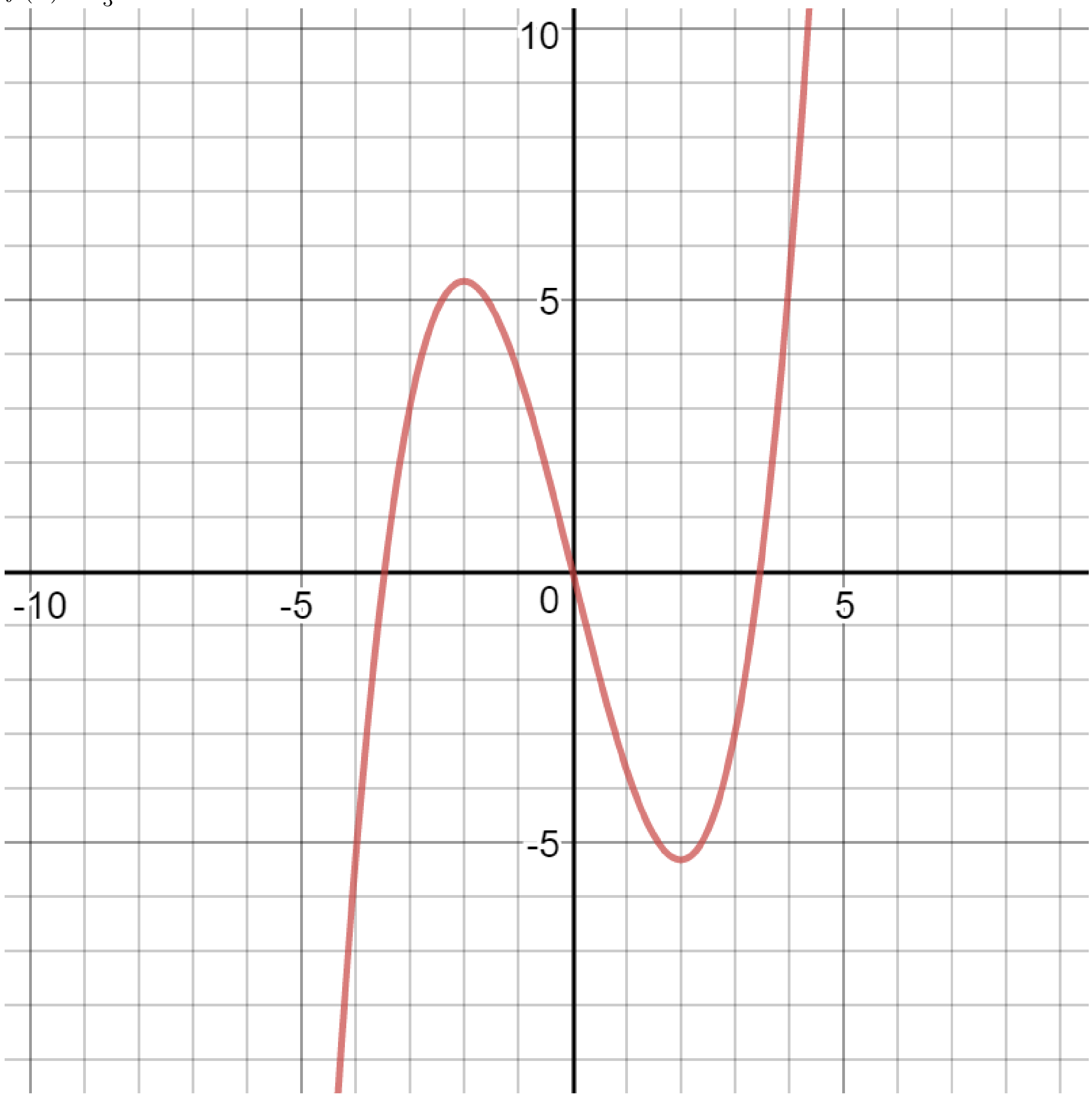
   (b) $f(x) = \left(x - \frac{3}{x}\right)^2$

9. By drawing small line segments tangent to the following graphs and calculating their slopes, sketch a graph of the derivative of each function:

   (a) $f(x) = x^2$



(b) $f(x) = \frac{1}{3}x^3 - 4x$



10. (Continuation) Calculate $f'(x)$ and $f''(x)$ for the following:

    (a) $f(x) = x^2$

    (b) $f(x) = \frac{1}{3}x^3 - 4x$

11. (Continuation) Using your solutions to the previous problems, answer the following questions:

    (a) What do you notice about the graph of f when $f'(x) = 0$?

    (b) What do you notice about the graph of f when $f'(x) < 0$?

    (c) What do you notice about the graph of f when $f'(x) > 0$?

    (d) For part (b), what do you notice about the graph of f when $f''(x) = 0$?

12. (*2016 Feb-March Paper 1*) Given: $f(x) = 2x^3 - 23x^2 + 80x - 84$
    (a) Prove that $(x - 2)$ is a factor of f.
    (b) Hence, or otherwise, factorise $f(x)$ fully.
    (c) Determine the x-coordinates of the turning points of f.
    (d) Sketch the graph of f, clearly labelling ALL turning points and intercepts with the axes.
    (e) Determine the coordinates of the y-intercept of the tangent to f that has a slope of 40 and touches f at a point where the x-coordinate is an integer.
13. (*2016 Feb-March Paper 1*) A soft drink can has a volume of 340 cm^3, a height of h cm and a radius of r cm.
    (a) Express h in terms of r.
    (b) Show that the surface area of the can is given by $A(r) = 2\pi r^2 + 680r^{-1}$.
    (c) Determine the radius of the can that will ensure the surface area is a minimum.

Finance, Growth and Decay
Geometric Series and Compound Interest

1. On 1 July 2012, you deposit 1000 rand into an account that pays 6% interest annually. How much is this investment worth on 1 July 2032?
2. (Continuation) On 1 July 2013, you deposit 1000 rand into an account that pays 6% interest annually. How much is *this* investment worth on 1 July 2032? Answer the same question for 1 July 2014, 1 July 2015, and so forth, until you see a pattern developing in your expressions.
3. (Continuation) Suppose that you deposit 1000 rand into the same account on 1 July *every* year. The problem now is to calculate the combined value of *all* these deposits on 1 July 2032, including the deposit made on the final day. Rather than getting the answer by tediously adding the results of *twenty-one* separate (but similar) calculations, we can find a shorter way. Let V stand for the number we seek, and observe that
$$V = 1000(1.06)^0 + 1000(1.06)^1 + 1000(1.06)^2 + \cdots + 1000(1.06)^{19} + 1000(1.06)^{20}$$
is the very calculation we wish to avoid. Obtain a second equation by multiplying both sides of this equation by 1.06, then find a way of combining the two equations to obtain a compact, easy-to-calculate formula for V.
4. (Continuation) Any list *first*, *first* $\times$ *multiplier*, *first* $\times$ *multiplier*^{2}, ..., in which each term is obtained by multiplying its predecessor by a fixed number is called a *geometric sequence*. A *geometric series*, on the other hand, is an addition problem formed by taking consecutive terms from some geometric sequence. For example, $32 - 16 + 8 - 4 + \cdots + 0,125$ is a nine-term geometric series whose sum is 21,375. Consider now the typical geometric series, which looks like *first*+*first* $\times$ *multiplier*+*first* $\times$ *multiplier*^{2}+ ... +*last*. Find a compact, easy-to-calculate formula for the sum of all these terms.

5. Sometimes it is necessary to invest a certain amount of money at a fixed interest rate for a fixed number of years so that a financial goal is met. The initial amount invested is called the *present value* and the goal is called the *future value*. The parents of an American child decide that \$350 000 will be needed for college expenses. They find a certificate of deposit that pays 0,5 percent interest each month. How much (present value) should they invest so that there is \$350 000 on the child's 18th birthday?

6. (Continuation) The parents realize they do not have that type of cash, so they decide they will deposit the same amount every month into the account, with their last payment on the child's 18th birthday. How much must they invest per month in order to have \$350 000 in the account on the child's 18th birthday?

7. Suppose that you invest R10 000 in an account that pays 0,5% interest per month. In how many months will the account have double your initial investment?

8. Nthoko invests R10 000 in a stock that returned 0,5% monthly interest for 2 years. Unfortunately, that stock took a nose dive and the value fell by 0,5% each month for two years. How much money did he have after the 4 years?

9. Would you rather have an account that promised 6% interest p.a., compounded annually, or an account that promised 0,5% monthly interest, compounded monthly?

10. (*2016 Feb-March Paper 1*) Diane invests a lump sum of R5 000 in a savings account for exactly 2 years. The investment earns interest at 10% p.a., compounded quarterly.
    (a) What is the quarterly interest rate for Diane's investment?
    (b) Calculate the amount in Diane's savings account at the end of the 2 years.

Repaying Loans

11. A bank has just granted Jordan a 10 000 rand loan, which will be paid back in 48 equal monthly installments, each of which includes a 1% monthly interest charge on the unpaid balance. The loan officer was amazed that Jordan (who knows about geometric series) had already calculated the monthly payment. Here's how Jordan figured it out:
    (a) Pretend first that the monthly payments are all R300. The first payment must include R100 just for *interest* on the R10 000 owed. The other R200 *reduces the debt*. That leaves a debt of R9800 after the first payment. Follow this line of reasoning and calculate the amount owed after four more payments of R300 have been made.
    (b) Some notational shorthand: Let A_n be the amount owed after n payments (so that $A_0 = 10000$), let $i = 0,01$ be the monthly interest rate, and let M be the monthly payment (which might not be R300).
        i. Explain why $A_1 = A_0 - (M - iA_0) = (1 + i)A_0 - M$.
        ii. Explain why $A_2 = (1 + i)A_1 - M$.

iii. Write an equation for A_n, the amount owed after n payments, in terms of A_{n-1}, the amount owed after one fewer payment.

(c) We wish to write A_n in terms of A_0. For example, you could write

$$\begin{aligned} A_2 &= (1+i)A_1 - M \\ &= (1+i)[(1+i)A_0 - M] - M \\ &= (1+i)^2 A_0 - (1+i)M - M \end{aligned}$$

i. How could we write A_3 in terms of A_0? You should see a pattern developing!

ii. How could we write A_n in terms of A_0? Your answer should involve the finite geometric series $M + (1+i)M + (1+i)^2 M + \cdots + (1+i)^{n-1}M$.

iii. Simplify your answer for the previous part by writing the finite geometric series in a more compact form.

(d) If N is the total number of monthly payments made, explain why $A_N = 0$ for Jordan's loan. Explain why $M = \dfrac{iA_0(1+i)^N}{(1+i)^N - 1}$.

(e) Calculate Jordan's monthly payment using the formula from (d). In Jordan's case, the monthly payment is less that R300.

12. What is the monthly payment needed to repay a R500 000 loan in 10 years, if the bank charges 0,8 percent per month?

13. In December, Thuli took out a R10 000 loan with monthly interest rate 0,7 percent. In order to pay back the loan, Thuli has been paying R871,70 a month since January. How many payments does Thuli need to pay everything back? How much does this loan actually cost her? What would the monthly payment have been if Thuli had been scheduled to pay back the loan in 24 months?

14. (*2016 Feb-March Paper 1*) Motloi inherits R800 000. He invests all of his inheritance in a fund which earns interest at a rate of 14% p.a., compounded monthly. At the end of each month, he withdraws R10 000 from the fund. His first withdrawal is exactly one month after his initial investment.

(a) How many withdrawals of R10 000 will Motloi be able to make from this fund?

(b) Exactly four years after his initial investment Motloi decides to withdraw all the remaining money in his account and use it as a deposit towards a house.

i. What is the value of Motloi's deposit, to the nearest rand?

ii. Motloi's deposit is exactly 30% of the purchase price of the house. What is the purchase price of the house, to the nearest rand?

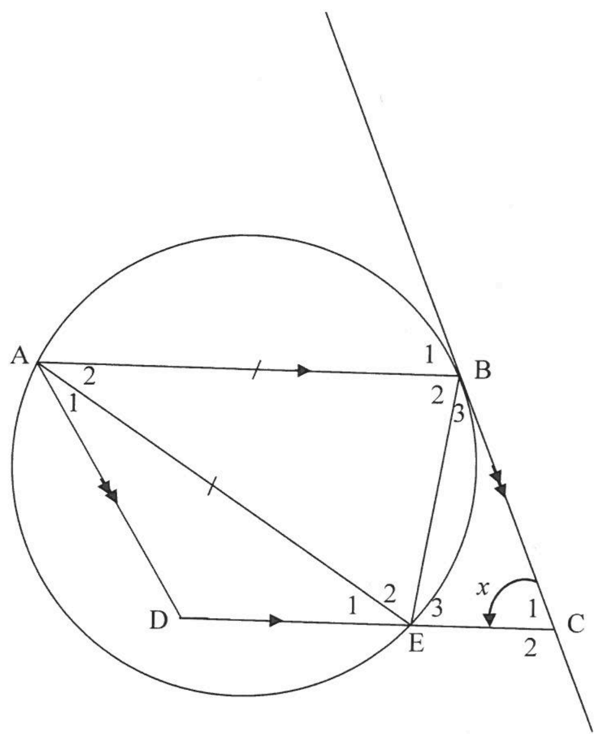
F.E.T. Phase

- Euclidian Geometry
- Probability

Euclidian Geometry

The following 3 problems are from the Senior Certificate Examinations Mathematics P2 from May/June 2016:

8.2 In the diagram below, the circle passes through A, B and E. ABCD is a parallelogram. BC is a tangent to the circle at B. AE = AB. Let $\hat{C}_1 = x$

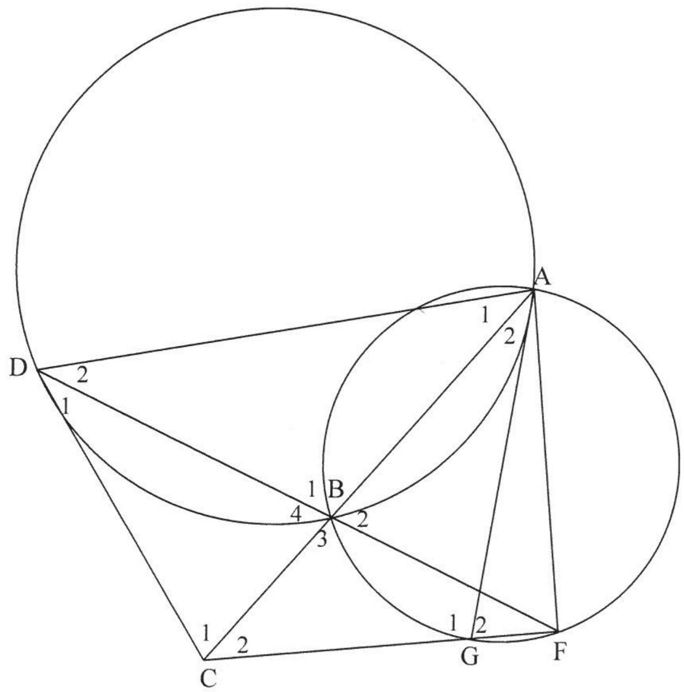


8.2.1 Give a reason why $\hat{B}_1 = x$ (1)

8.2.2 Name, with reasons, THREE other angles equal in size to x. (6)

8.2.3 Prove that ABED is a cyclic quadrilateral. (3)

9.2 In the diagram below, two unequal circles intersect at A and B. AB is produced to C such that CD is a tangent to the circle ABD at D. F and G are points on the smaller circle such that CGF and DBF are straight lines. AD and AG are drawn.



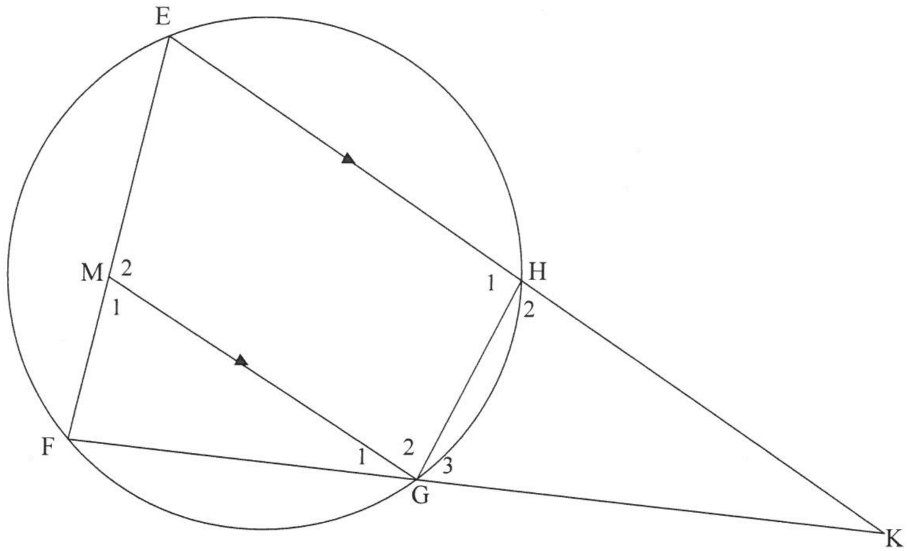
Prove that:

9.2.1 $\hat{B}_4 = \hat{D}_1 + \hat{D}_2$ (4)

9.2.2 AGCD is a cyclic quadrilateral (4)

9.2.3 DC = CF (4)

10.2 In the diagram below, cyclic quadrilateral EFGH is drawn. Chord EH produced and chord FG produced meet at K. M is a point on EF such that MG | | EK. Also KG = EF



10.2.1 Prove that:

(a) $\Delta KGH \mid\mid\mid \Delta KEF$ (4)

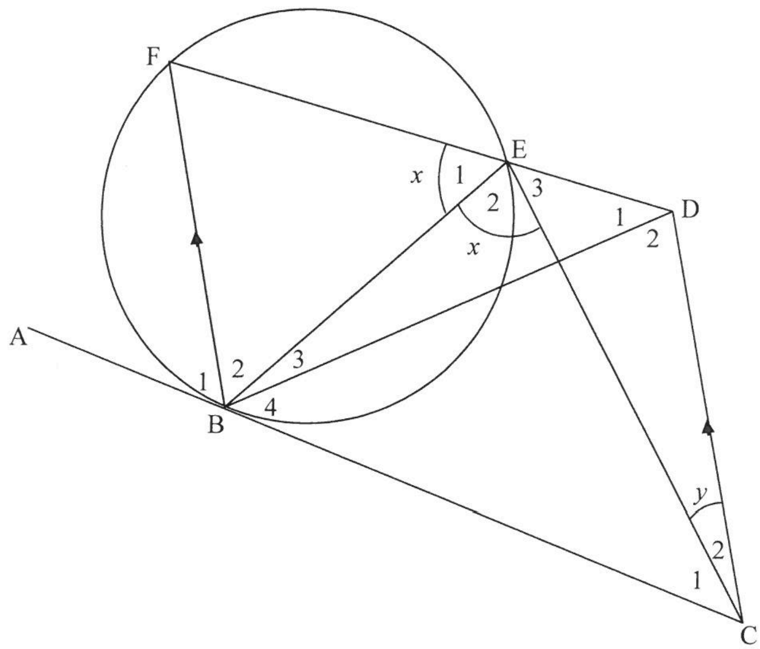
(b) $EF^2 = KE \cdot GH$ (2)

(c) $KG^2 = EM \cdot KF$ (3)

10.2.2 If it is given that KE = 20 units, KF = 16 units and GH = 4 units, calculate the length of EM. (3)

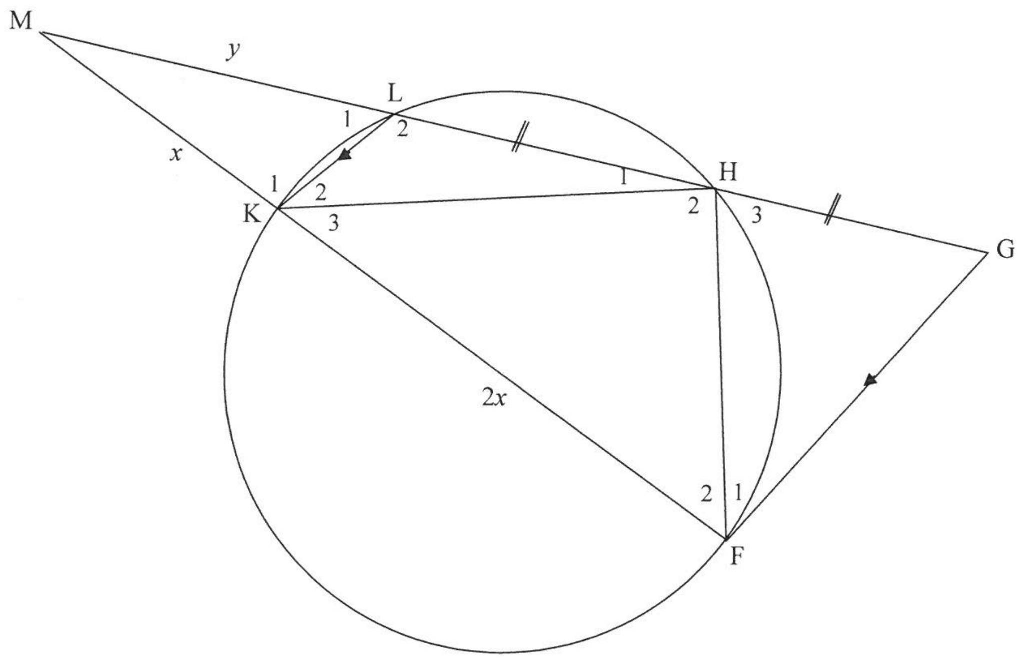
The following 2 problems are from the Senior Certificate Examinations Mathematics P2 from November 2016:

ABC is a tangent to the circle BFE at B. From C a straight line is drawn parallel to BF to meet FE produced at D. EC and BD are drawn. $\hat{E}_1 = \hat{E}_2 = x$ and $\hat{C}_2 = y$.



9.1	Give a reason why EACH of the following is TRUE:		
	9.1.1	$\hat{B}_1 = x$	(1)
	9.1.2	$B\hat{C}D = \hat{B}_1$	(1)
9.2	Prove that BCDE is a cyclic quadrilateral.		(2)
9.3	Which TWO other angles are each equal to x?		(2)
9.4	Prove that $\hat{B}_2 = \hat{C}_1$.		(3)

10.2 In the diagram HLKF is a cyclic quadrilateral. The chords HL and FK are produced to meet at M. The line through F parallel to KL meets MH produced at G. $MK = x$, $KF = 2x$, $ML = y$ and $LH = HG$.



10.2.1 Give a reason why $G\hat{F}M = L\hat{K}M$. (1)

10.2.2 Prove that:

(a) $GH = y$ (3)

(b) $\Delta MFH \,|||\, \Delta MGF$ (5)

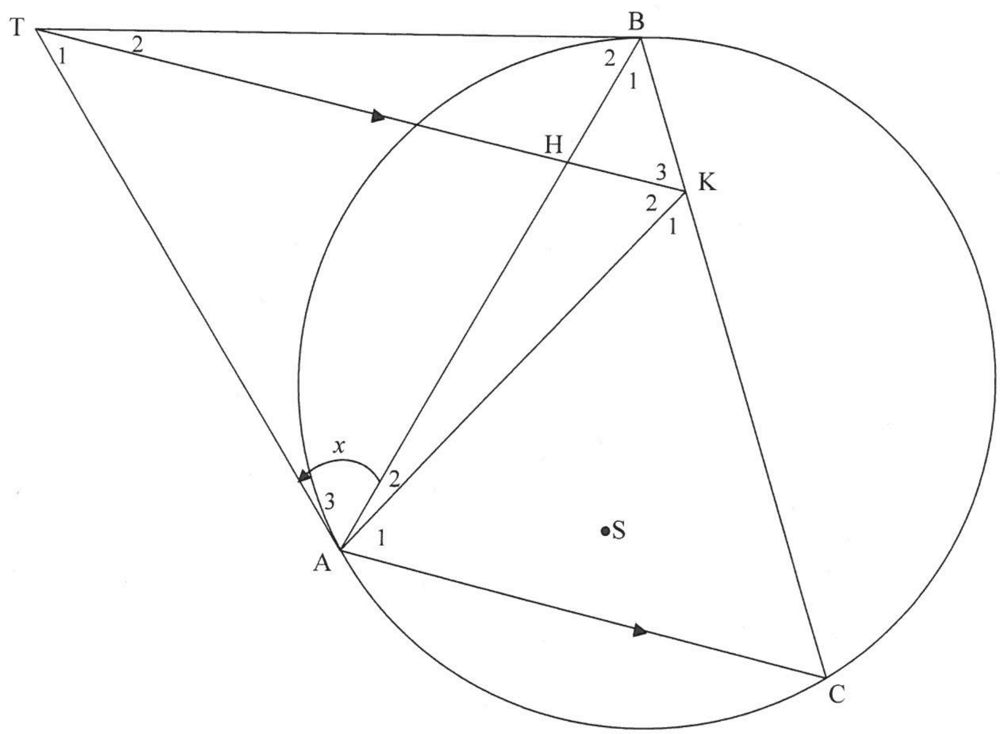
(c) $\dfrac{GF}{FH} = \dfrac{3x}{2y}$ (2)

10.2.3 Show that $\dfrac{y}{x} = \sqrt{\dfrac{3}{2}}$ (3)

The following problem is from the Senior Certificate Examinations Mathematics P2 from November 2015:

QUESTION 9

In the diagram below, ΔABC is drawn in the circle. TA and TB are tangents to the circle. The straight line THK is parallel to AC with H on BA and K on BC. AK is drawn. Let $\hat{A}_3 = x$.

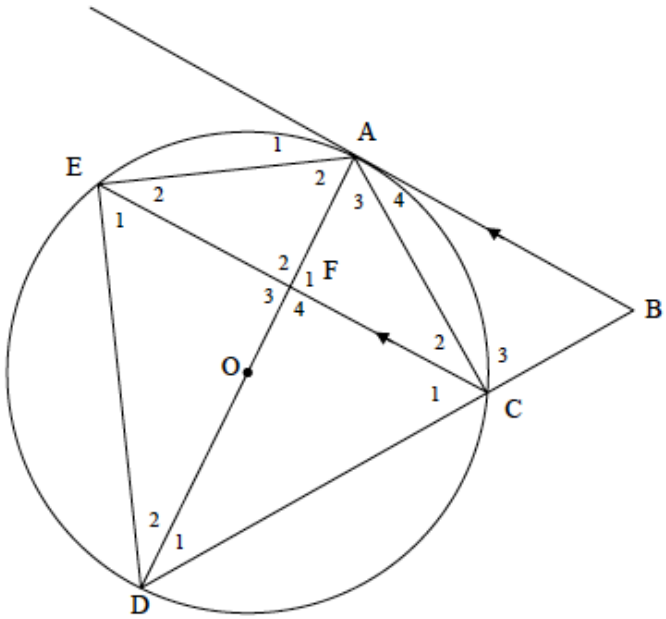


9.1	Prove that $\hat{K}_3 = x$.	(4)
9.2	Prove that AKBT is a cyclic quadrilateral.	(2)
9.3	Prove that TK bisects $A\hat{K}B$.	(4)
9.4	Prove that TA is a tangent to the circle passing through the points A, K and H.	(2)
9.5	S is a point in the circle such that the points A, S, K and B are concyclic. Explain why A, S, B and T are also concyclic.	(2)

The following problem is from the Senior Certificate Examinations Mathematics P3 from November 2009:

QUESTION 11

In the figure below, AB is a tangent to the circle with centre O. AC = AO and BA || CE. DC produced, cuts tangent BA at B.



11.1	Show $\hat{C}_2 = \hat{D}_1$.	(3)
11.2	Prove that $\Delta ACF \parallel\!| \Delta ADC$.	(3)
11.3	Prove that $AD = 4AF$.	(4) **[10]**

Probability

The following problem is from the Senior Certificate Examinations Mathematics P1 from November 2015:

QUESTION 11

11.1 For two events, A and B, it is given that:

P(A) = 0,2
P(B) = 0,63
P(A and B) = 0,126

Are the events, A and B, independent? Justify your answer with appropriate calculations. (3)

11.2 The letters of the word DECIMAL are randomly arranged into a new 'word', also consisting of seven letters. How many different arrangements are possible if:

11.2.1 Letters may be repeated (2)

11.2.2 Letters may not be repeated (2)

11.2.3 The arrangements must start with a vowel and end in a consonant and no repetition of letters is allowed (4)

11.3 There are t orange balls and 2 yellow balls in a bag. Craig randomly selects one ball from the bag, records his choice and returns the ball to the bag. He then randomly selects a second ball from the bag, records his choice and returns it to bag. It is known that the probability that Craig will select two balls of the same colour from the bag is 52%.

Calculate how many orange balls are in the bag. (6)
[17]

The following problem is from the Senior Certificate Examinations Mathematics P1 from November 2016:

QUESTION 12

The digits 1 to 7 are used to create a four-digit code to enter a locked room. How many different codes are possible if the digits may not be repeated and the code must be an even number bigger than 5 000? **[5]**

The following problem is from the Senior Certificate Examinations Mathematics P1 from May/June 2016:

QUESTION 10

10.1 A tournament organiser conducted a survey among 150 members at a local sports club to find out whether they play tennis or not. The results are shown in the table below.

	PLAYING TENNIS	NOT PLAYING TENNIS
Male	50	30
Female	20	50

10.1.1 What is the probability that a member selected at random is:

(a) Female (2)

(b) Female and plays tennis (1)

10.1.2 Is playing tennis independent of gender? Motivate your answer with the necessary calculations. (3)

10.2 The probability of events A and B occurring are denoted by P(A) and P(B) respectively.

For any two events A and B it is given that:

- $P(B') = 0{,}28$
- $P(B) = 3P(A)$
- $P(A \text{ or } B) = 0{,}96$

Are events A and B mutually exclusive? Justify your answer. (4)

[10]

QUESTION 11

Five boys and four girls go to the movies. They are all seated next to each other in the same row.

11.1 One boy and girl are a couple and want to sit next to each other at any end of the row of friends. In how many different ways can the entire group be seated? (3)

11.2 If all the friends are seated randomly, calculate the probability that all the girls are seated next to each other. (3)

The following problem is from the Senior Certificate Examinations Mathematics P3 from November 2008:

QUESTION 5

5.1 The Matric Dance Committee has decided on the menu below for the 2008 Matric Dance. A person attending the dance must choose only ONE item from each category, that is starters, main course and dessert.

MENU

STARTERS	MAIN COURSE	DESSERT
Crumbed Mushrooms	Fried Chicken	Ice-cream
Garlic Bread	Beef Bolognaise	Malva Pudding
Fish	Chicken Curry	
	Vegetable Curry	

5.1.1 How many different meal combinations can be chosen? (2)

5.1.2 A particular person wishes to have chicken as his main course. How many different meal combinations does he have? (2)

5.2 A photographer has placed six chairs in the front row of a studio. Three boys and three girls are to be seated in these chairs.

In how many different ways can they be seated if:

5.2.1 Any learner may be seated in any chair (2)

5.2.2 Two particular learners wish to be seated next to each other (3)

F.E.T. Phase

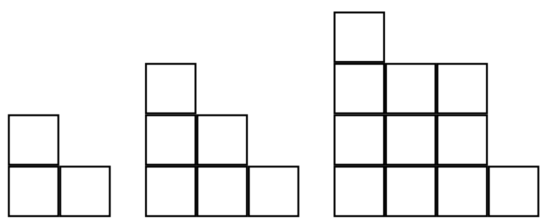
Algebra: Problem solving and quadratic inequalities;
Trigonometry: Introduction, excursions, graphs, 3D trigonometry, identities

Algebra

Problem Solving Strategies

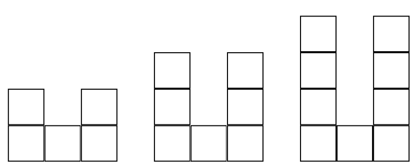
1. Act it out
2. Draw a diagram
3. Make a table
4. Make a graph
5. Work backwards
6. Systematize the counting process
7. Look for a pattern
8. Find a rule
9. Make the problem simpler

Square steps



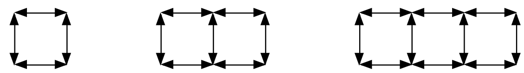
STEP 1 STEP 2 STEP 3

"U"



Step 1 Step 2 Step 3

Toothpicks 1

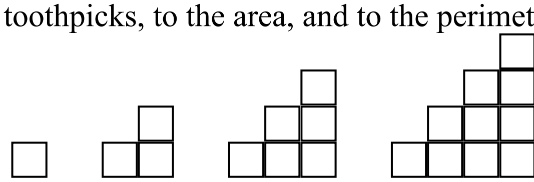


1. How many squares are in step n?
2. How many toothpicks are in step n?
3. How many horizontal toothpicks are in step n?
4. How many vertical toothpicks are in step n?
5. How many rectangles are in step n? (Recall that a square is a rectangle.)

Note: Learners may only be able to get a recursive formula $t_n = t_{n-1} + n$, not an explicit formula, $t_n = \frac{n(n+1)}{2}$.

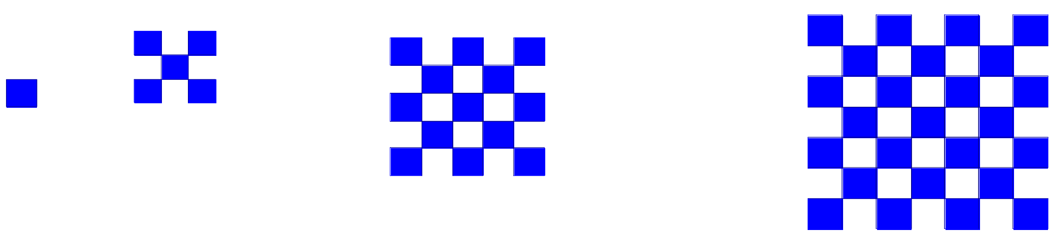
Toothpicks 2

The shapes shown below are made with toothpicks. Find rules that relate the shape number to the number of toothpicks, to the area, and to the perimeter.



Checkerboard numbers

The first four checkerboard numbers are 1, 5, 13, and 25. Tile representations are shown below. What is the 50^{th} checkerboard number? What is the n^{th} checkerboard number?



Proportion Solves "Work" Problems

To the number of methods that can be used to solve "work" problems we add one more. Consider the problem of determining the time needed to finish a job if, working alone, John needs 2 hours and Mary needs 3 hours to complete the job.

John does 1 job in 2 hours or ½ job/hour and Mary does 1 job in 3 hours or 1/3 job/hour. Thus, we have (1/2 + 1/3) = 5/6 job done in hour. We then need only solve the proportion 5/6 job/1 hour = 1 job/x hours. Cross multiplying and solving gives x = 6/5 hours, or 1 hour 12 minutes.

In general, if one does a job in a hours and another in b hours when working alone, then we have the proportion (1/a + 1/b) job/1 hour = 1 job/ x hours. This is just another way to think about the problem.

Quadratic Inequalities

Solve the following:

1. $x^2 \le 16$
2. $(x-3)(x+5) > 0$
3. $x^2 - 3x - 10 \le 0$
4. $x^2 - x > 6$
5. Write the steps needed to solve a quadratic inequality.
6. $x^2 + 9 \ge 0$
7. $(\mathrm{x}-3)^2 \le 0$

Additional Problems:

8. $|x-3| \le 5$
9. $\mathrm{x}(\mathrm{x}-4)(\mathrm{x}+5) \ge 0$
10. $x^3 - 3x^2 - 10\mathrm{x} \le 0$
11. $\dfrac{\mathrm{x}(\mathrm{x}-4)}{\mathrm{x}+5} \le 0$

Problems from Past Matric Exams:

1. $x^2 = 5x - 4$ (3)
2. $x(3 - x) = -3$ (5)
3. $3 - x < 2x^2$ (5)
4. $|5 + x| > 3$ (4)
5. $x^2 + 7x < 0$ (3)
6. Solve for x:

   $\dfrac{8}{|x+2|} < 4 \quad (x \ne -2)$ (6)

7. Solve for x: $|4 - x| \le 20$ (5)

Past Remarks from Examiner's Report on Specific Concerns

Algebra:

- Absolute value. Many candidates experienced difficulty with the absolute value inequality $|4-\mathrm{x}| \le 20$ (Many candidates left out the $|\ |$ symbol.)

Trigonometry

I. Introduction to Trigonometry

Unit Circle
Find the exact value of tan 60˚ using your calculator. Find sin 60˚.

II. Excursions in Trigonometry

1. Three stakes on the ground mark points A, B and C, the vertices of a triangle. Find the area of the triangle using more than one method.
2. One stake marks point A on one side of the street. The base of a tree is at point B on the other side of the street. Without crossing the street, find the distance between A and B. Do this in two ways and compare your results.
3. a.) Find the height of a building near you.
   b.) Without crossing the street, find the height of a building across the street.
4. Two stakes in the ground mark point A and point B on opposite sides of a wall between the points. Find the shortest distance between A and B.
5. Two stakes in the ground mark point A and point B on opposite sides of a wall between the points. A third stake at point C is on the ground in the same line with the wall. Find the area of triangle ABC.
6. Two stakes in the ground mark point A and point B on opposite sides of a wall between the points. A third stake at point C is on the ground in the same line with the wall. Find the distance between A and B.
7. An ant is at point A on one wall and she wants to walk to her home at point B that is located on a wall that is perpendicular to the wall where she is.
   a. What is the shortest distance she must walk to get home?
   b. At what angle relative to a horizontal line must she walk?
8. An ant is at point A on one wall and she wants to walk to her home at point B that is located on the opposite side of the wall where she is.
   a. What is the shortest distance she must walk to get home?
   b. At what angle relative to a horizontal line must she walk?
9. a. Without opening your box, find the length of the longest diagonal.
   b. A concrete box is 3 m by 5 m and 10 m high. Find the length of the longest diagonal.
10. One person must get on the ground and sight to the top of the pole. A second person will then move the metre stick until the observer's line of sight includes the top of the metre stick and the top of the pole. Then measure the distance from the observer's eye to the bottom of the metre stick.

    Draw a picture of this and decide which trig function you can use to find the angle of elevation.

    What are possible sources of error in this method?

    How would you determine the height of the pole?

    How can you use the metre stick and shadows to determine the height of the pole?

III. Trigonometric Graphs

Problems from Past Matric Exams:

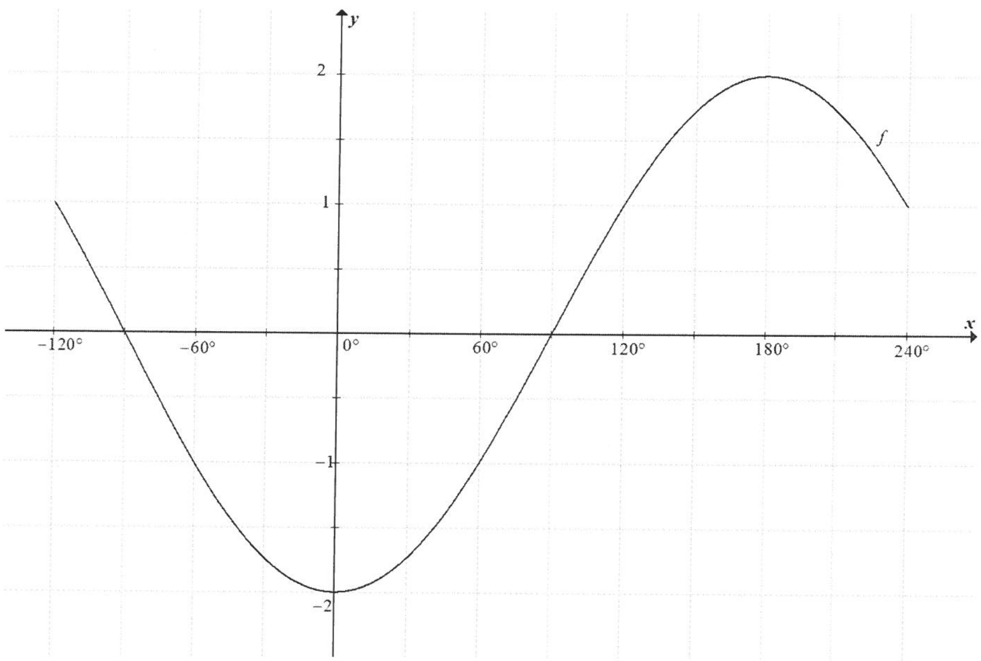
QUESTION 6

Given the equation: $\sin(x + 60°) + 2\cos x = 0$

6.1 Show that the equation can be rewritten as $\tan x = -4 - \sqrt{3}$. (4)

6.2 Determine the solutions of the equation $\sin(x + 60°) + 2\cos x = 0$ in the interval $-180° \leq x \leq 180°$. (3)

6.3 In the diagram below, the graph of $f(x) = -2\cos x$ is drawn for $-120° \leq x \leq 240°$.



6.3.1 Draw the graph of $g(x) = \sin(x + 60°)$ for $-120° \leq x \leq 240°$ on the grid provided in the ANSWER BOOK. (3)

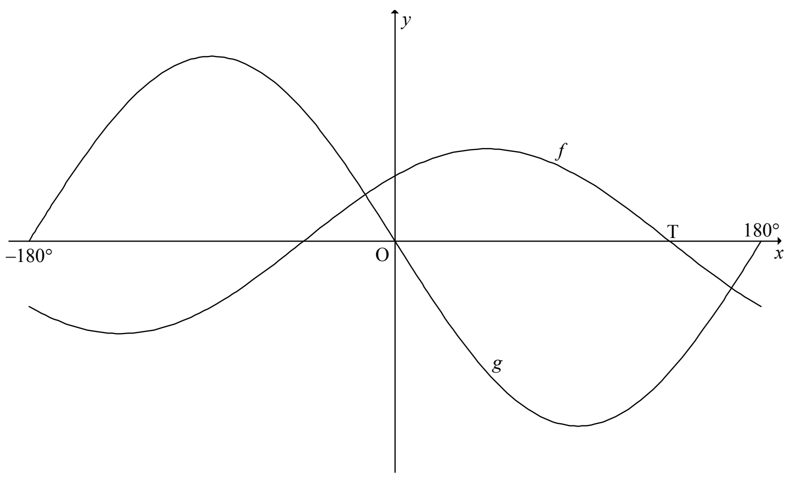
6.3.2 Determine the values of x in the interval $-120° \leq x \leq 240°$ for which $\sin(x + 60°) + 2\cos x > 0$. (3)

[13]

Metz

QUESTION 9

The graphs of $f(x) = \cos(x - 45°)$ and $g(x) = -2\sin x$ are drawn below for $x \in [-180°\,;180°]$.
The point T is an x-intercept of f as indicated on the diagram.



9.1	Show that $\cos(x - 45°) = -2\sin x$ can be written as $\tan x = -0{,}2612$.	(4)
9.2	Solve the equation: $\cos(x - 45°) = -2\sin x$ for $x \in [-180°\,;180°]$.	(3)
9.3	Write down the coordinates of point T.	(2)
9.4	Write down the interval for which $f(x) \geq g(x)$.	(2)
9.5	Write down the interval for which both f and g are strictly increasing.	(3)
9.6	The graph h is obtained when the graph f is shifted 45° to the right. Write down the equation of h in its simplest form.	(2) **[16]**

4.1 On the same set of axes, draw the graphs of f and g where
$f : x \rightarrow 2\tan x$ and $g : x \rightarrow \sin 2x$ for $x \in [0°; 180°]$ (7)

4.2 Write down the range of g. (2)

4.2 Use the graphs to indicate, with the symbols A, B, C, D … on your graph, where the values of x of the following can be read off:
$g(x) - f(x) = 0$ (4)

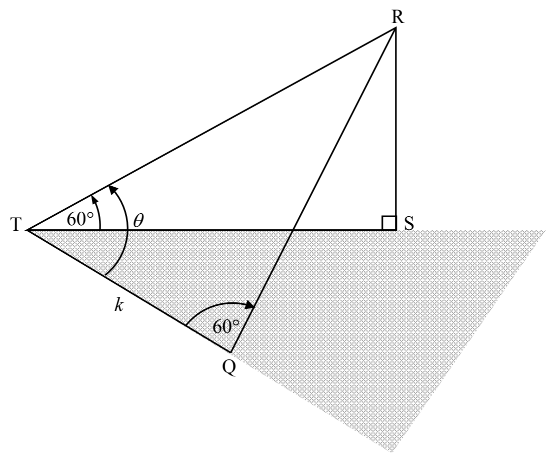
[13]

IV. 3 Dimensional Trigonometry

Problems from Past Matric Exams:

QUESTION 10

In the diagram below, RS is the height of a vertical tower. T and Q are two points in the same horizontal plane as the foot S of the tower. From point T the angle of elevation to the top of the tower is 60°. $R\hat{T}Q = \theta$, $R\hat{Q}T = 60°$ and TQ = k metres.

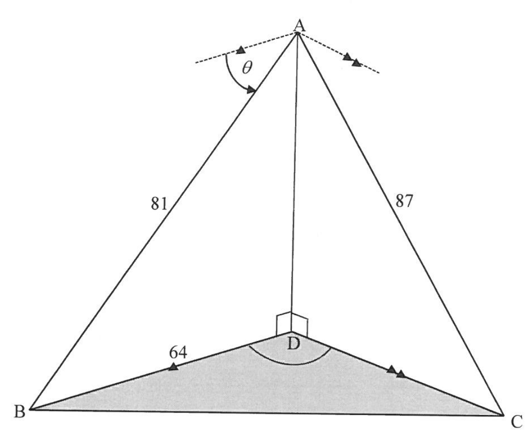


10.1 Express TR in terms of θ and k. (3)

10.2 Show that $RS = \dfrac{3k}{2(\sqrt{3}\cos\theta + \sin\theta)}$. (7)
[10]

QUESTION 7

From point A an observer spots two boats, B and C, at anchor. The angle of depression of boat B from A is θ. D is a point directly below A and is on the same horizontal plane as B and C. BD = 64 m, AB = 81 m and AC = 87 m.



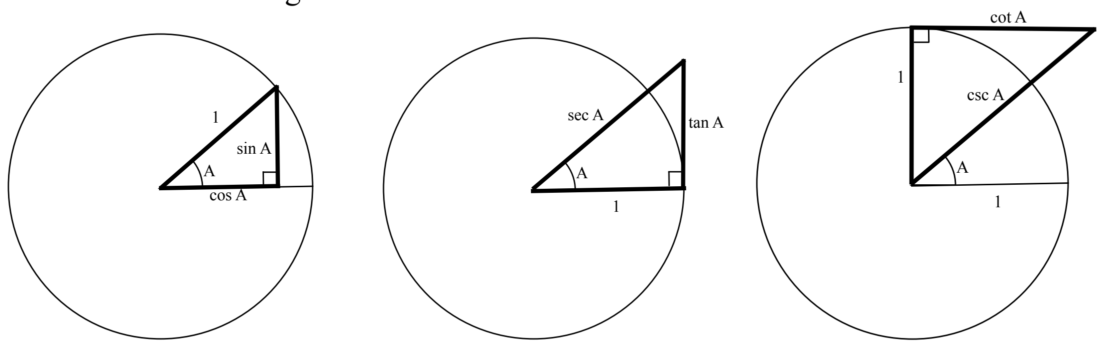
7.1 Calculate the size of θ to the nearest degree. (3)

7.2 If it is given that $B\hat{A}C = 82{,}6°$, calculate BC, the distance between the boats. (3)

7.3 If $B\hat{D}C = 110°$, calculate the size of $D\hat{C}B$. (3)
[9]

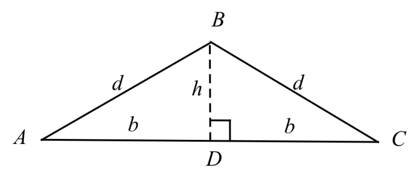
V. Identities

1. Three nice triangles



Write the Pythagorean Theorem for each of the three triangles.

2. Half-angle Formulas from an Isosceles Triangle



a.) Use the figure to find $\sin\left(\frac{B}{2}\right)$.

b.) Use the Law of Cosines, to find $2b$; then find b/d.

c.) Use your results to write a formula for $\sin\left(\frac{B}{2}\right)$.

Suggestions for proving identities

(1) Do not assume LHS=RHS e.g. by cross multiplying
(2) Using the basic identities, simplify the more complicated expression OR
(3) Rewrite both expressions in terms of Sine and Cosine
(4) Avoid taking square roots or raising to powers if possible.
(5) Note any restrictions on the variable
(6) Take all the arguments of the sines and cosines and find the greatest common divisor of them. Use factoring. Difference of two squares very useful in factoring. Check for identities after each step
(7) Put each side over a common denominator.
(8) Whenever second or higher powers of cos (u) occur, use the identity $\cos^2(u) = 1 - \sin^2(u)$ to convert them to powers of sin(u).
(9) Simplify whenever you can by combining like terms and pulling out common factors. Multiply by 1.
(10) When you have obtained an obvious identity, reverse all the steps to start with the obvious identity and end with what you want to show. This will be the proof of your identity.

Problems from Past Matric Exams:

1. Prove the identity: $\dfrac{2\tan x - \sin 2x}{2\sin^2 x} = \tan x$ (6)

2. a.) Simplify $\dfrac{4\sin x\cos x}{2\sin^2 x - 1}$ to a single trigonometric ratio.(3)

   b.) Hence, calculate the value of $\dfrac{4\sin 15^\circ \cos 15^\circ}{2\sin^2 15^\circ - 1}$ WITHOUT using a calculator.
   (Leave your answer in simplest surd form.) (2)

3. Determine the value of $\dfrac{1}{\cos(360^\circ - \theta)\cdot\sin(90^\circ - \theta)} - \tan^2(180^\circ + \theta)$ (6)

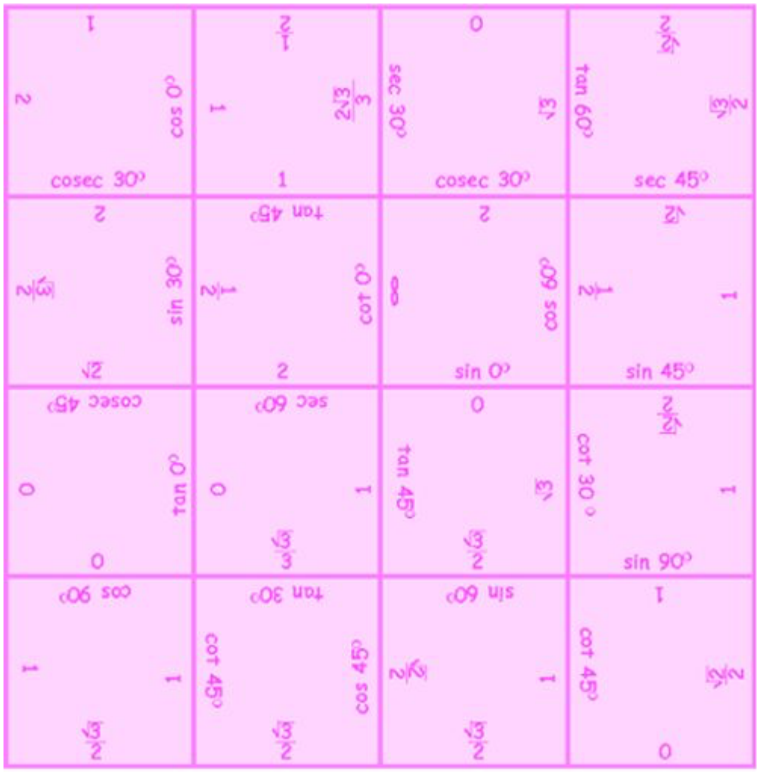
4. If $\sin x - \cos x = \dfrac{3}{4}$, calculate the value of $\sin 2x$ WITHOUT using a calculator. (5)

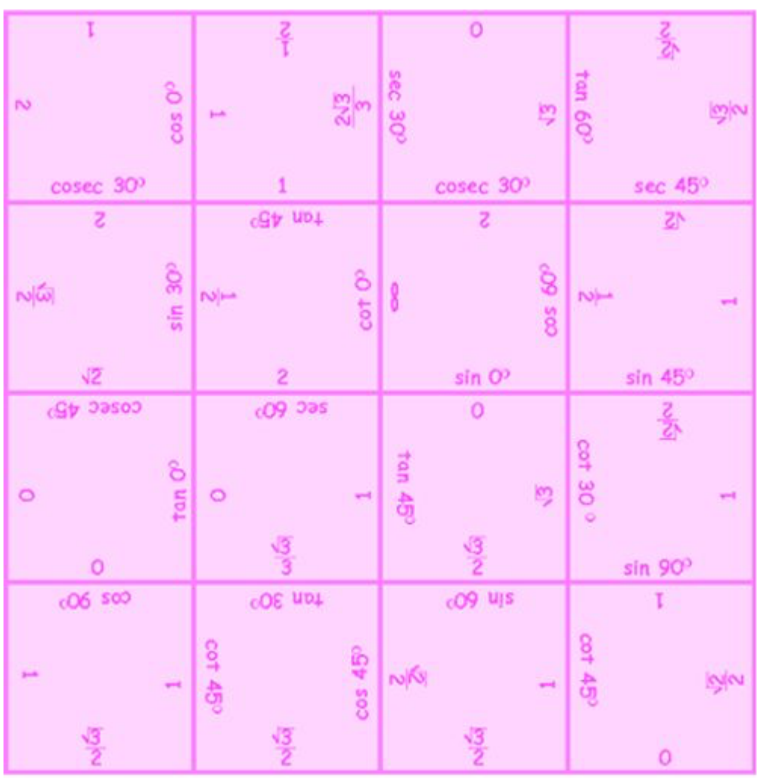
5. Prove the following identity: $\dfrac{\cos^2\left(90^\circ + \theta\right)}{\cos\left(-\theta\right) + \sin\left(90^\circ - \theta\right)\cos\theta} = \dfrac{1}{\cos\theta} - 1$ (6)

6. Consider the following expression: $2\sin^2 3x - \sin^2 x - \cos^2 x$
   a.) Simplify the expression to a single trigonometric ratio of x. (3)
   b.) Write down the maximum value of the expression. (1)

7. It is given that $p = \cos\alpha + \sin\alpha$ and $q = \cos\alpha - \sin\alpha$
   Determine the following trigonometric ratios of p and/or q:
   a.) $\cos 2\alpha$ (3)
   b.) $\tan\alpha$ (4)
   c.) Simplify $\dfrac{p}{2q} - \dfrac{q}{2p}$ to a single trigonometric ratio of α. (6)

Puzzle: Cut along the lines, shuffle, then put together again. Recommendation: Copy first so you have a key.





FUNCTIONS AND INVERSES

I. Introduction

Mapping and functions are common in our everyday language. We may understand how maps work, and how to get from point A to B, but then why do we need the inverse? (To go in the opposite direction!) Can you act out a function and its inverse in space? There are several ways we are going to discuss functions and their inverses – numerically, graphically, and algebraically. We even have special notation to denote functions and their inverses.

II. Definitions

a. A **function** is a correspondence or pairing between 2 variables such that each value of the first (independent) variable corresponds to exactly one value of the second (dependent) variable.
    i. A **function** is a relation in which no different ordered pairs have the same first coordinate.
    ii. A function is **one-to-one** if every element of the range corresponds to exactly one element of the domain. This happens when the function passes both the vertical and horizontal line test. We will get more into this later…
    iii. Can you come up with a real-life analogy for a function?
b. **Domain** – the set of all possible x-values (i.e. inputs, independent variables) over which a function is defined
c. **Range** – the set of all y-values (i.e. outputs, dependent variables) of a function
d. **Inverse function** – a function that "reverses" another function
    i. See "notes" throughout the document

III. Numerical Relations

Which of the following are functions? For those that are functions, state the domain and range. If they are not functions, explain why.

a) $\{(2, 3), (2, 5), (2, 6), (2, 7)\}$

b) $\{(2, 3), (2, 3), (2, 3), (2, 3)\}$

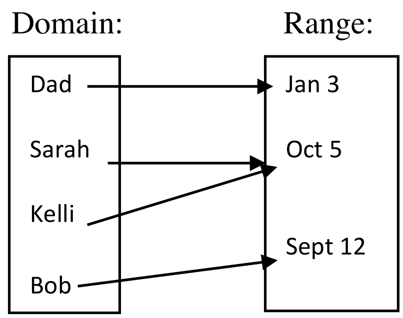
c) $\{(2, 3), (3, 5), (5, 4), (4, 2)\}$

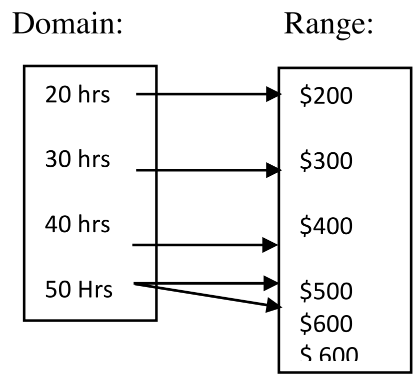
d) $\{(2, 3), (3, 3), (4, 3), (5, 3)\}$

In a table, check to see that every *x* there is a unique *y*-value. Tell whether the following tables represent functions.

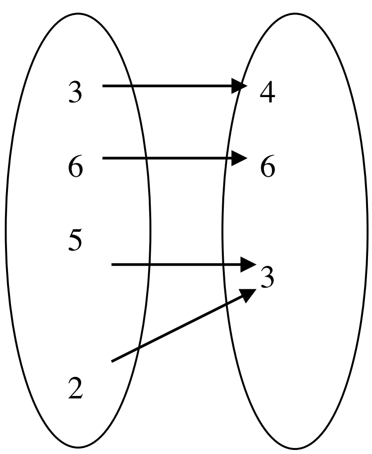
x	*y*
1	2
3	4
4	2

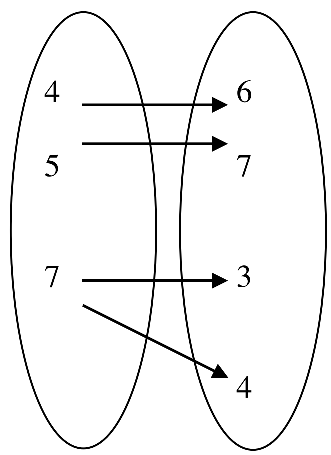
x	*y*
3	5
4	5
3	6





In a mapping check to see that every *x* is mapped to a unique *y*-value. Tell whether the mapping is a function or not.

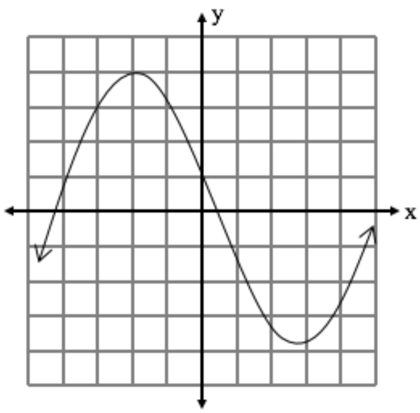


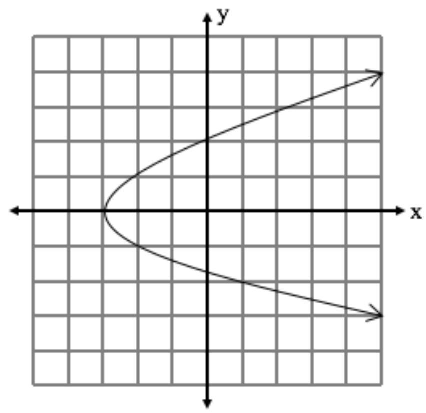


Note: For inverses, the x and y-values switch. In other words, the domain and range switch. The input becomes the output and the output becomes the input. For example, if the point (1,2) is on the original function, then the point (2,1) is on its inverse.

IV. Graphical Relations

In graphs, we can implement the "vertical line test" to determine if a relation is a function. If no vertical lines intersect a graph in no more than one point, then it is a function. Conversely, if it does, then the graph is not a function. Which of the following graphs are functions?





Note: Because the x and y-values of functions and their inverses switch, their graphs are symmetrical across the line y=x. Try it out!

V. Algebraic Relations

Based on the definition of a function, determine which of the following are functions.

$Y = 6x - 2$	$y^2 = 6x - 2$
$Y^3 = 6x - 2$	$y^4 = 6x - 2$
$Y = 5/x$	$x = y^2 - 6$
$x^3 = y - 3$	$x^2 = 2y + 1$
$2^x = y + 1$	$y = \sqrt{x}$

Note: Because the x and y-values of functions and their inverses switch, in order to create the equation of a function's inverse, replace x with y and vice versa, then solve the equation for y. Try it below!

Find the inverse of the following functions.

$Y = 6x - 2$	$y^2 = 6x - 2$

VI. Function Notation and Evaluating Functions

f(x) notation: The symbol f(x) is read "f of x". (The parenthesis does not stand for multiplication, but rather enclose the independent variable.) The letter *f* is the "name" of the function. Answer the following questions.

If $g(x) = 2x^2 - 5$ what is the "name" of the function? _____

If $g(x) = 2x^2 - 5$, then $g(2) =$ _______ If $g(x) = 2x^2 - 5$, then $g(-3) =$ ______

If $g(x) = 2x^2 - 5$, then $g(c) =$ ________ If $f(x) = 2^x$, then $f(2) =$ ______

If $f(x) = 2^x$, and $f(x) = 16$, then $x =$ _____

Complete the following given $f(x) = 3 / x$ and $g(x) = \frac{3x - 2}{x}$

a) $f(-1) =$ _________ b) $g(1) =$ __________

c) $f(\,0\,) =$ __________ d) $g(\,q\,) =$ ________________

e) $f(\,g(0)\,) =$ ________ f) $g\,(g\,(-1)\,) =$ _______________

Use the following graph of f(x) and g(x) to fill in the blanks:

a) $f(1) =$ ______

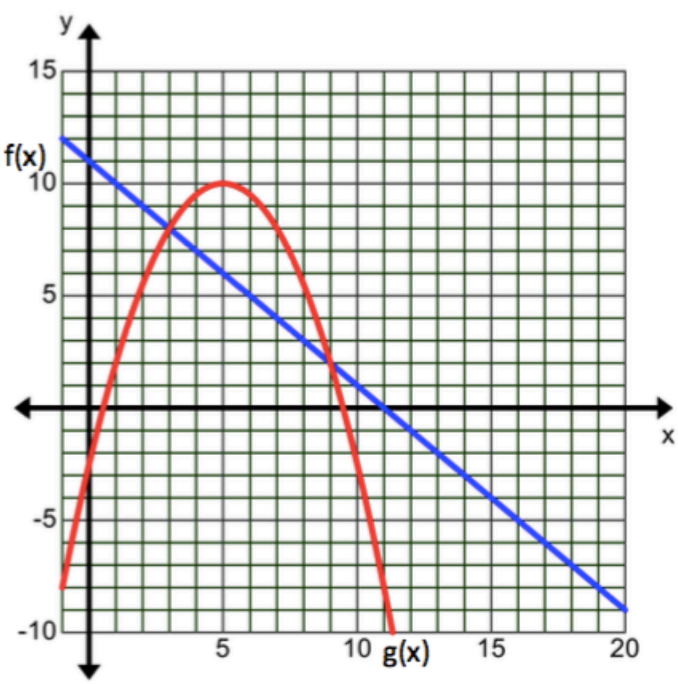
b) g(____ or ____) = 2

c) $f\,(g\,(8)) =$ ________

d) If $f(a) = g(a)$, then $a =$ ______

e) $h(\,-2\,) = a$, $a =$ _______

f) $h\,(\,b\,) = 4$, $b =$ _______



VII. Modeling linear functions

1. Flying Otter Delivery Service (FODS) charges $6.25 to send an 8-ounce envelope from Johannesburg to East London a distance of 1,000 km. FODS charges $4.00 to send the same envelope from Johannesburg to Bloemfontein, which is 400 km away.

a) Create a linear equation for *(km*, *cost*).

b) How much will it cost to send an 8-ounce envelope from Johannesburg to Cape Town, if Cape Town is 1,400 km away?

c) Yunus has $20.17. How far can he send his 8-ounce envelope using FODS delivery service?

2. Suppose you own a car that is 40 months old. You use the internet to determine that the value of the car is now $24,000. You also remember checking on the value of the car 10 months ago and at that time the value was $32,000.

a) Find a linear equation that you can use to predict the value of your car. (*Month, Value$$$*)

b) What is the meaning of the slope of this equation? What does it tell you about the value of this car over time? (Explain what the slope means in the context of this problem.)

c) You decide to sell the car when the value is $4,800. How old will the car be at this point in time?

d) What was the value of the car when it was brand new?

VIII. Classroom Activities

Activity 1: Barbie Bungee

Hook: Watch a short youtube video about bungee jumping

Question: How is Barbie's vertical drop related to the length of the rubber band chain?

Materials: Barbies (or small stuffed animals, dolls, etc.), rubber bands, tape measurer
Optional materials: whiteboard/chalkboard, marker/chalk, computer access

Setup: Wrap one rubber band around Barbie's feet, then attach subsequent rubber bands using slip knots. Use the table below to record Barbie's vertical drop as you add more and more rubber bands to the chain.

Questions to start:

1. What do you predict will be the relationship between the number of rubber bands and Barbie's vertical drop? Why?
2. Estimate: How far do you think Barbie will fall if she is attached to a chain of 20 rubber bands?
3. Which of these is the independent and which is the dependent variable? How can you tell?

Gather data:

Number of rubber bands	Vertical drop (cm)

Analyze data:

1. Educated guess: How far do you think Barbie will fall if she is attached to a chain made up of 20 rubber bands?
2. Let's create a more precise model. Open https://www.desmos.com/calculator and add a table. Enter your data. Sketch your scatterplot below, labeling both axes.
3. What type of function does your data appear to best represent? Why does this model make sense here?
4. Pick two representative points and find a linear equation through them. Show your work below.
5. Graph your equation on the same set of axes on Desmos. How well does this equation fit your data points?
6. Rewrite your equation more generally as $y = ax + b$ and click "all" to create sliders. Adjust your sliders until you feel that the line fits the data well (click on the number 10 if you need to adjust the range of the sliders). What is your final equation?

Interpret Data:

7. Where does your graph cross the y-axis? What does this point mean in the context of this lab?
8. What is the slope of your line? What does this value mean in the context of this lab?

9. Calculate: What does your model predict will be Barbie's vertical drop if she were attached to a chain of 20 rubber bands? Does this make sense? Was it close to your initial guess?
10. Ask another group for their line of best fit for their data.
    1. What does their equation mean?
    2. How and why is it different than yours?
    3. How could you use their model to predict Barbie's vertical drop if she were attached to a chain of 50 rubber bands?

Reflection:

11. What potential sources of error could have affected the accuracy of your model?
12. What would you tweak if you did this lab again?
13. If your group is given 10 points to divide up in any way that you think is fair, how many points would you allocate to each member of the group? Why? What was your role in the group?

CULMINATING COMPETITION!

Which group can ensure that Barbie will safely fall from a height of ________, while still enjoying the thrill of getting as close to the ground as possible?

Create your equation (or inequality) here and show all work that led to your answer.

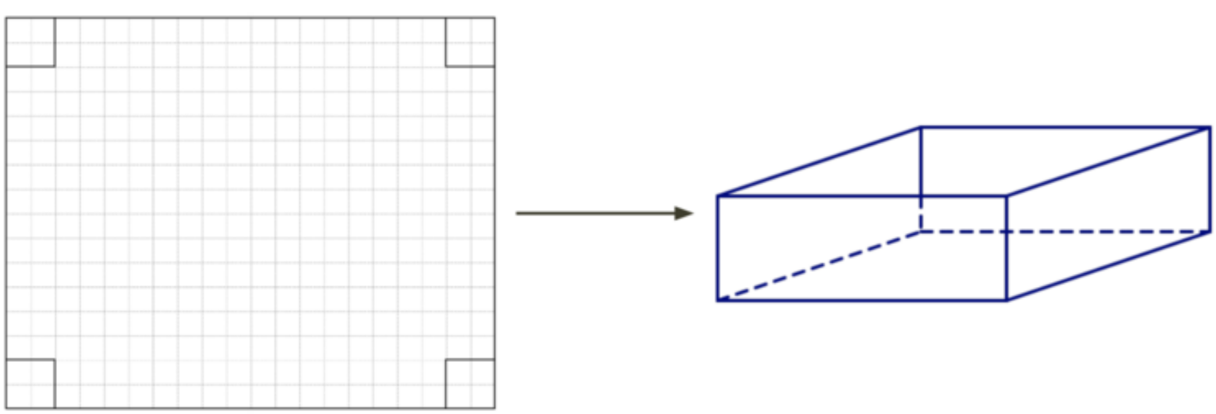
Activity 2: Popcorn Cuboid

Hook: You get to eat as much popcorn fits in your box!

Question: How can you maximize the volume of your cuboid in order to get the most popcorn?

Materials: Popcorn, 8.5 x 11in paper, tape, scissors, ruler, Desmos or graphing calculator

Setup: Cut out squares from the corners of your paper, then fold the edges up to create an open top box.



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Questions to start:

1. What do you predict to be the optimal size of the cut squares to create a cuboid of the largest volume?
2. Which of these is the independent and which is the dependent variable? How can you tell?
3. How many pieces of popcorn do you predict to fit in your open top box?

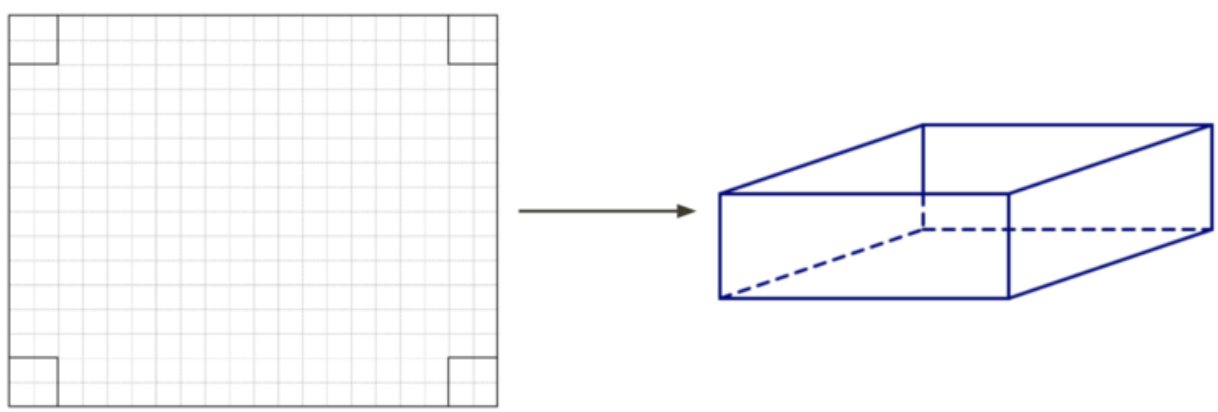
Gather data:

Test it out! Cut squares from the corners of your paper and build your cuboid.

Dimensions of your cut squares:
Dimensions of your cuboid:
Volume of your cuboid:

Analyze and Interpret data:

1. What equation did you use to find the volume of your cuboid?
2. How can you generalize that equation to create an equation of the volume of any cuboid? (Hint: If you do not know the dimensions of the cut out squares, how should you represent that length?) It may help to label the lengths of the sides of the diagram below, then create your function.



3. What type of function did you create? Write it in standard form in the space below.
4. What does the x-axis represent? What does the y-axis represent?
5. How many zeros does your function have?
6. What are the zeros? (And what are their multiplicities?)
7. What do your zeros represent in the context of this problem? Does this make sense? Why or why not?

Graph Your Data:

8. Graph your function on Desmos.com and check to see if your zeros are accurate. Sketch the graph below, labeling all intercepts and extrema (maximum/minimum).
9. Notice what happens to your graph when it reaches the zeros. Does the curve cross the x-axis at your zeros or does it "bounce off" the zeros, like what happens with the graph of $f(x)=x^2$.
10. Looking at your graph, what dimensions of the cut squares will give you the largest volume of the cuboid?

11. How close were you to your prediction?
12. What is the maximum volume of the cuboid?
13. How many pieces of popcorn were in your cuboid? How close were you to your prediction?

Follow up Questions:

14. How could you calculate how many pieces of popcorn are in a given cuboid? What information would you need to know?
15. How could you run this task differently next time?
16. What other types of mathematical questions could I ask you about this modeling task?

Activity 3: Time to Draw!

Hook: Doodling in math class!

Question: How can you use a combination of five different families of functions (linear, quadratic, cubic, exponential,

Materials: Graph paper and pencil/colored pencils/pens
Optional materials: Computer with online access to Desmos.com

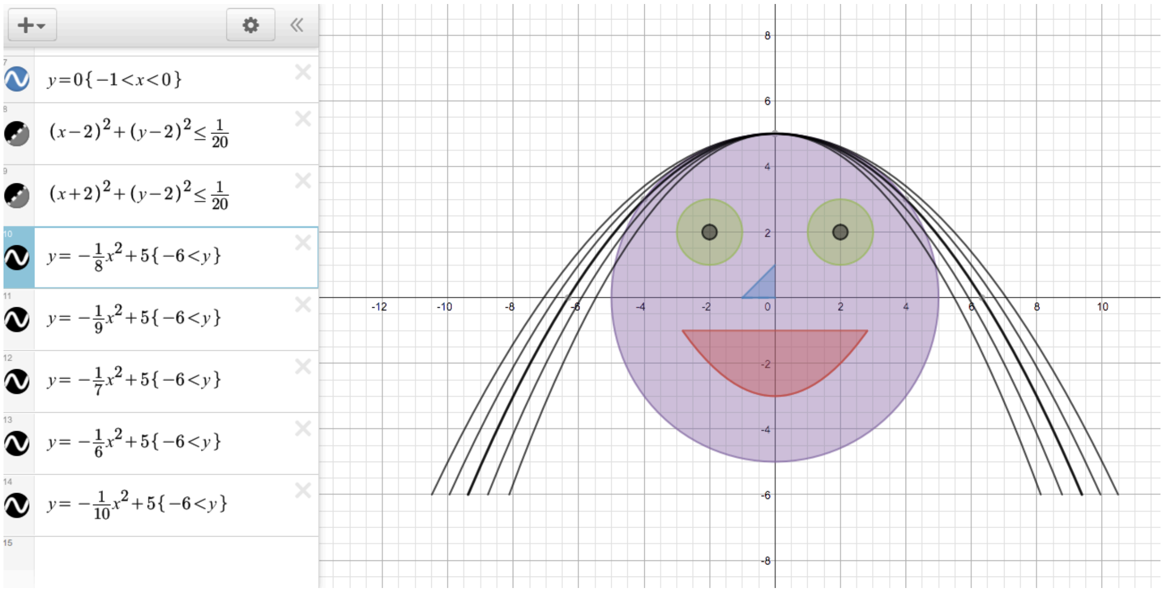
Setup: At a minimum, familiarize yourself with the graphs of linear, quadratic, cubic, and exponential functions. Think of a funny face or simple stick figure person to draw by hand.

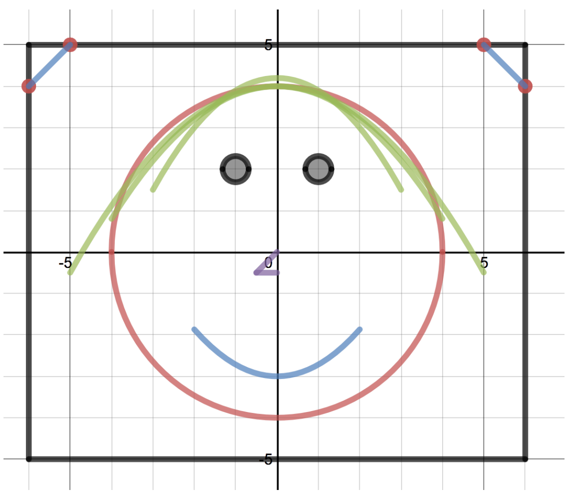
Linear functions:	$y = mx + b$ $y - y_1 = m(x - x_1)$ $Ax + By = C$	ex: y = x
Quadratic functions:	$y = ax^2 + bx + c$ $y - k = a(x - h)^2$	ex: $y = x^2$
Cubic functions:	$y = ax^3 + bx^2 + cx + d$	ex: $y = x^3$
Exponential functions:	$y = a \cdot b^x$	ex: $y = 2^x$
Circles/Ellipses:	$(x - h)^2 + (y - k)^2 = r^2$	ex: $x^2 + y^2 = 1$

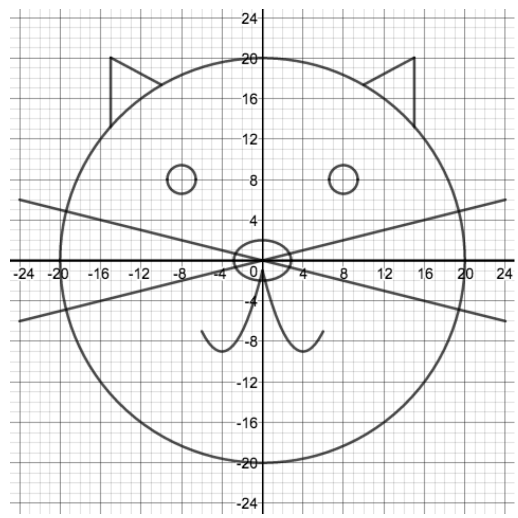
Questions to Start: How might you be able to use graphs of different functions to create a funny face or stick figure? What function would resemble the mouth? The eyes? Hair? Nose?

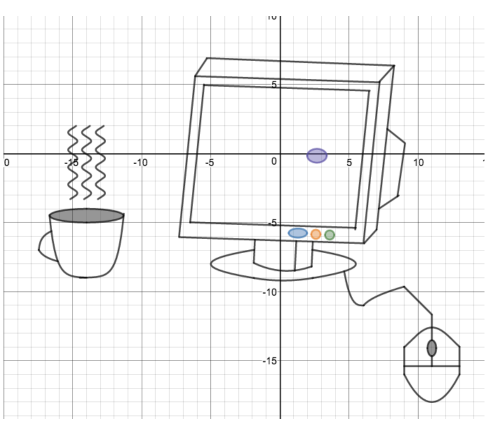
Task: On your graph paper, or on your computers, use a variety of functions to create the image that you desire. You must restrict the domain and/or range to prevent the graphs of your functions from continuing infinitely in both directions. Be sure to label which equation corresponds to which part of the image.

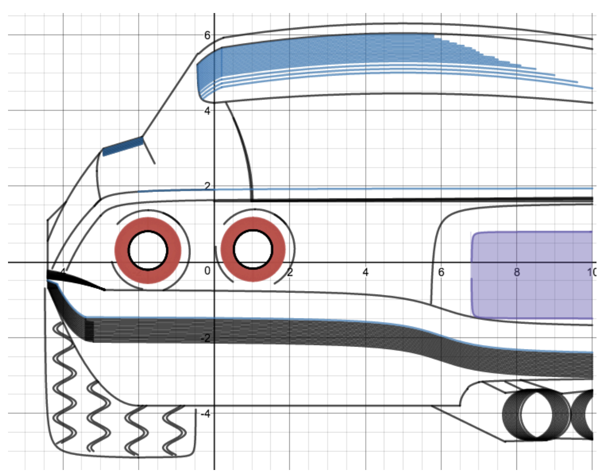
Here are some examples of images that were created on Desmos.com.











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